Math. 350-04
Spring, 2018
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Final Exam

Tuesday, May 1, 2018, 8:00AM to 10:30AM

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. (Do it carefully, and check it.) Each entire problem is worth 25 points.

1. A tank contains 300 gallons of water and 15 pounds of salt. Water containing a salt concentration of .125 pounds per gallon flows into the tank at a rate of 3 gallons per minute, and the mixture in the tank flows out at the same rate.
   (a) Write down a differential equation relating the quantity Q of salt in the tank at time t and the derivative of Q.
   (b) Solve that differential equation.
   (c) Determine the amount of salt in the tank after a long time.

2. Consider the initial value problem

$$y'' + 8y' + 17y = \sin(t), \quad y(0) = 1,\ y'(0) = 0.$$

   (a) Solve the initial value problem. Be sure to carefully write down all steps in your solution process.
   (b) Write down the steady-state solution.

3. Write down the degree 5 polynomial approximation to the series solution to the following initial value problem.

$$y' + x^2 y = 0, \quad y(0) = -1.$$

4. Use Laplace transforms to solve the following initial value problem. You may use the table on the back of this sheet.

$$y''+4y'+3y = f(t), \quad y(0) = 0,\ y'(0) = 1, \quad \text{where } f(t) = \left\{ \begin{array}{cc} 0, & 0 \le t < 2 \\ -1, & 2 \le t < 3 \\ 0, & t \ge 3 \end{array} \right\}.$$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. t^n, $n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 7
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}$, $n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 19
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28