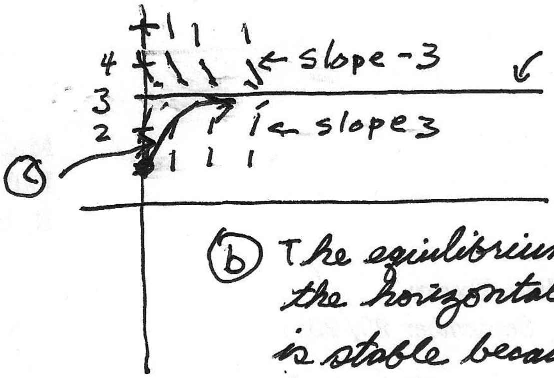
① (a)



(b) The equilibrium is $y=3$, corresponding to the horizontal asymptote. The equilibrium is stable because $a<0$ when the equation is written in the form $y'=ay+b$.

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② We first put the equation into the form $y'+py=q$:

$y'+\left(\frac{1}{t}+1\right)y=\frac{2}{t}$. An integrating factor is then:

$\mu=e^{\int 1+\frac{1}{t}\,dt}=e^{t+\ln(t)}=te^t$. We then have:

$(te^ty)'=2e^t$, whence $te^ty=2e^t+C$,

whence $y(t)=\frac{2}{t}+\frac{C}{te^t}$. Plugging in the initial value then gives:

$1=\frac{2}{1}+\frac{C}{e}\Rightarrow$

$\frac{C}{e}=-1\Rightarrow C=-e\Rightarrow y(t)=\frac{2}{t}-\frac{e}{te^t}=\frac{1}{t}\left(2-e^{-t+1}\right)$

3) $\frac{dv}{dt} = 9.8 - \frac{1}{10}v,\ v(0)=0.$

We first solve the IVP:

$$\frac{dv}{dt} = \frac{-1}{10}(v-98) \Rightarrow \int \frac{1}{v-98}\frac{dv}{dt}\,dt = \int \frac{-1}{10}\,dt$$

$$\Rightarrow \ln|v-98| = \frac{-1}{10}t + c \Rightarrow v(t) = 98 + Ce^{\frac{-1}{10}t}$$

$$v(0) = 0 = 98 + C \Rightarrow C = -98 \Rightarrow \boxed{v(t) = 98\left(1-e^{\frac{-1}{10}t}\right)}.$$

a) The time in seconds would be given by:

$$T:\ v(T) = (.75)(98) = 98\left(1-e^{-\frac{1}{10}T}\right)$$

$$\Rightarrow e^{-\frac{1}{10}T} = (.25)\cancel{(98)} \Rightarrow -\frac{1}{10}T = \ln[(.25)\cancel{(98)}]$$

$$\Rightarrow T = -10\ln[(.25)\cancel{(98)}] \approx 10\ln(4) \approx \boxed{13.9 \text{ seconds}}$$

b) The total distance is

$$\int_0^T v(t)\,dt = \int_0^T 98\left(1-e^{-\frac{1}{10}t}\right)dt = 98\left(t + 10e^{-\frac{1}{10}t}\right)\Big|_{t=0}^{T}.$$

$$\approx 98\left[\left(13.9 + 10e^{-1.39}\right) - 10\right] \approx \boxed{626 \text{ meters}}.$$