1 (a) $y = \sum_{n=0}^{\infty} a_n x^n$. $y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$, $y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$.

$$y'' = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n.$$

$$(x+1)y' = x\sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$= \sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

$$xy = x\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n x^{n+1} = \sum_{n=1}^{\infty} a_{n-1} x^n.$$

Thus, $y'' + (x+1)y' + xy$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + \sum_{n=1}^{\infty} a_{n-1} x^n$$

$$= 2a_2 + a_1 + \sum_{n=1}^{\infty} \left[(n+2)(n+1)a_{n+2} + n a_n + (n+1)a_{n+1} + a_{n-1}\right] = 0.$$

Thus, for $n=0$, we have:

$a_2 = -\frac{1}{2} a_1$, and $a_{n+2} = \frac{-1}{(n+2)(n+1)}\left[(n+1)a_{n+1} + n a_n + a_{n-1}\right]$ for $n \geq 1$.

Thus, $a_0 = 1$, $a_1 = 0$, $a_2 = 0$,

$n=1$: $a_3 = \frac{-1}{6}\left[2a_2 + a_1 + a_0\right] = \frac{-1}{6}\left[0 + 0 + 1\right] = \frac{-1}{6}$

$n=2$: $a_4 = \frac{-1}{12}\left[3a_3 + 2a_2 + a_1\right] = \frac{-1}{12}\left[\frac{-3}{6} + 0 + 0\right] = \cancel{\frac{1}{20}} \cdot \frac{3}{72} = \frac{1}{24}$

~~Thus: $y(x)$~~

$n=3$: $a_5 = \frac{-1}{20}\left[4a_4 + 3a_3 + a_2\right] = \frac{-1}{20}\left[\frac{1}{6} - \frac{1}{2} + 0\right] = \frac{-1}{20}\left[\frac{-1}{3}\right] = \cancel{\frac{2}{100}}\ \frac{1}{60}$

Thus, $\boxed{y(x) \approx 1 - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{60}x^5}$

(b) The initial value problem can be transformed to the IVP for part (a) through the substitution $t = x+1$. The degree 5 polynomial is thus

$$\boxed{y(x) = 1 - \frac{1}{6}(x-1)^3 + \frac{1}{24}(x-1)^4 + \frac{1}{60}(x-1)^5}$$