1. $y=\sum_{n=0}^{\infty} a_n x^n$, $xy' = x\sum_{n=1}^{\infty} n a_n x^{n-1} = \underline{\sum_{n=1}^{\infty} n a_n x^n}$.

$(x+1)y = x\sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n x^{n+1} + \sum_{n=0}^{\infty} a_n x^n$

$= \underline{\sum_{n=1}^{\infty} a_{n-1} x^n + \sum_{n=0}^{\infty} a_n x^n}$.

$y'' = \underline{\sum_{n=2}^{\infty} (n+2)(n+1) a_{n+2} x^n}$.

Thus, $y'' + xy' + (x+1)y =$

$\sum_{n=1}^{\infty} n a_n x^n + \sum_{n=1}^{\infty} a_{n-1} x^n + \sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$

$= \cancel{a_1 x} + \cancel{a_0 x} + a_0 + \overset{2a_2}{\cancel{a_1 x}} + \sum_{n=1}^{\infty} \left[n a_n + a_{n-1} + a_n + (n+2)(n+1) a_{n+2}\right] x^n = 0.$

Thus: ($n=0$) $a_0 + 2a_2 = 0 \Rightarrow a_2 = -\frac{1}{2} a_0$

$\underline{n \geq 1}$: $a_{n+2} = -\frac{1}{(n+2)(n+1)} \left[(n+1) a_n + a_{n-1}\right]$

$a_0 = 1$, $a_1 = 0$, ($n=0$) $a_2 = \frac{-1}{2}$.

($n=1$) $a_3 = -\frac{1}{6}\left[2a_1 + a_0\right] = -\frac{1}{6} = a_3$

($n=2$) $a_4 = -\frac{1}{12}\left[3a_2 + a_1\right] = \frac{-1}{12}\left[\frac{-3}{2}\right] = \frac{1}{8} = a_4$

($n=3$) $a_5 = -\frac{1}{20}\left[4a_3 + a_2\right] = \frac{-1}{20}\left[\frac{-4}{6} - \frac{1}{2}\right] = \frac{-1}{20}\left[\frac{-7}{6}\right] = \frac{7}{120}$

Thus, $\boxed{y(x) \approx 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{8}x^4 + \frac{7}{120}x^5}$

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2. Problem 2 can be transformed to problem 1 with the substitution $t = X+1$. Thus, the required polynomial is:

$$\boxed{y(x) \approx 1 - \frac{1}{2}(x-1)^2 - \frac{1}{6}(x-1)^3 + \frac{1}{8}(x-1)^4 - \frac{7}{120}(x-1)^5}$$