Math. 350-01, Summer, 2015, second exam answers

① $W(y_1,y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{vmatrix}$

$= \cosh^2(t) - \sinh^2(t)$

$= \frac{1}{4}(e^t + e^{-t})^2 - \frac{1}{4}(e^t - e^{-t})^2$

$= \frac{1}{4}(e^{2t} + 2 + e^{-2t}) - \frac{1}{4}(e^{2t} - 2 + e^{-2t}) = \boxed{1}.$

② The characteristic equation is $r^2+6r+9=0$

$\therefore (r+3)^2=0$, so the general solution to the homogeneous equation is: $y_h(t) = c_1 e^{-3t} + c_2 t e^{-3t}$.

Thus, for undetermined coefficients, we choose a particular solution of the form $y_p(t) = At^2 e^{-3t}$.

We obtain: $y_p'(t) = 2Ate^{-3t} - 3At^2e^{-3t}$,

$y_p''(t) = 2Ae^{-3t} - 6Ate^{-3t} - 6Ate^{-3t} + 9t^2e^{-3t}$

$= 2Ae^{-3t} - 12Ate^{-3t} + 9At^2e^{-3t}$.

Plugging into the equation gives:

$$\begin{array}{rl} & 2Ae^{-3t} - 12Ate^{-3t} + 9At^2e^{-3t} \\ + & \cancel{1}2Ate^{-3t} - 18At^2e^{-3t} \\ + & 9At^2e^{-3t} \\ \hline = & 2Ae^{-3t} = e^{-3t}, \text{ so } 2A = 1, \boxed{A = 1/2}. \end{array}$$

Thus, the general solution is

$y_g(t) = c_1 e^{-3t} + c_2 t e^{-3t} + \frac{1}{2} t^2 e^{-3t}$.

$y_g(0) = \boxed{0 = c_1 \cancel{+ c_2}}$

$y_g'(t) = -3c_1e^{-3t} + c_2e^{-3t} - 3c_2te^{-3t} + te^{-3t} - \frac{3}{2}t^2e^{-3t}$

so $y_g'(0) = \boxed{-3c_1 + c_2 = 0}$, so $c_1 = c_2 = 0$,

so $\boxed{y(t) = \frac{1}{2} t^2 e^{-3t}}$