Math. 362-02
Fall, 2016
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Final Exam

Tuesday, December 6, 2016, 2:00PM to 4:30PM

This exam is closed book, but you may use calculators that do not have computer algebra systems. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner: You will be graded on what you show, in addition to your answer. Check your work carefully. Each entire problem is worth 20 points.

1. Consider the following system of equations

$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & + & 3x_3 & + & 4x_4 & = & 1, \\ -x_1 & + & x_2 & - & x_3 & + & x_4 & = & 0, \\ & & 3x_2 & + & 2x_3 & + & 5x_4 & = & 0, \\ 2x_1 & + & x_2 & + & 4x_3 & + & 3x_4 & = & 2. \end{array}$$

   (a) Write down the augmented matrix for this system of equations.

   (b) Use Gaussian elimination with back substitution to transform this system of equations into reduced row echelon form. **Show your computations!**

   (c) Interpret the reduced row echelon form in terms of the solution set to the system of equations. That is,

      (i) If there are no solutions, state so.

      (ii) If there is a unique solution, give it.

      (iii) If there is a parametrized solution set, write it down.

2. Consider $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

   (a) Find a basis for the null space of A.

   (b) Find a basis for the column space of A.

   (c) Find a basis for the row space of A.

3. Consider the following two bases.

$$B = \left\{ \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \right\} \quad \text{and} \quad B' = \left\{ \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}, \begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} \right\}.$$

   (a) Find the transition matrix $P_{B \to B'}$.

   (b) Compute the coordinate vector $[w]_{B'}$, where $[w]_B = \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix}$.

   (c) Compute the representation of w in the standard basis.

   (Hint: There is a simple exact answer that is not too hard to obtain without a calculator. However, be very careful in your computations.)

4. Compute the eigenvalues and corresponding eigenspaces of the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$.

5. For A as in Problem 4, write down a matrix P such that PAP^{-1} is a diagonal matrix.