Final Exam
Tuesday, May 13, 2003, 7:30AM

This exam is closed book. Make sure your name is on all pages. You should put your work on your own paper, and you may keep this exam sheet upon leaving. Be sure to check your work carefully, and to show intermediate computations in a logical presentation. **Check your calculations carefully.** Each entire problem is worth 16 points, and 4 points are free.

1. Solve the following system of equations by putting the corresponding augmented matrix in reduced row echelon form.

$$
\begin{array}{rcrcrcl}
x_1 & + & x_2 & + & x_3 & = & 6 \\
-x_1 & + & x_2 & + & x_3 & = & 4 \\
-x_1 & - & x_2 & + & x_3 & = & 0
\end{array}
$$

2. Write down an LU factorization of the matrix for the system in problem 1. Solve the system with this factorization.

3. Compute the eigenvalues and eigenvectors of A, where

$$
A = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}.
$$

4. Using the eigenvalues and eigenvectors of A as above, write down a decomposition of the form $A = VDV^{-1}$, then use that decomposition to compute A^4.

5. Compute a basis for the null space and write down a basis for the range of the following matrix.

$$
\begin{pmatrix}
2 & 1 & 3 & 3 \\
1 & 1 & 1 & 3 \\
0 & 0 & 2 & 0
\end{pmatrix}
$$

6. Compute a QR factorization of the matrix

$$
\begin{pmatrix} 6 & 16 \\ 8 & 18 \end{pmatrix}.
$$