

Second Exam

Tuesday, March 11, 2003

This exam is closed book. Make sure your name is on all pages. You should put your work on your own paper, and you may keep this exam sheet upon leaving. Be sure to check your work carefully, and to show intermediate computations in a logical presentation. **Check your calculations carefully.** Each entire problem is worth 25 points

1. Test the following set of vectors for linear independence. Show all computations.

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ 6 \\ 7 \\ 8 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 3 \\ 5 \\ 3 \\ 5 \end{pmatrix}.$$

2. What is the dimension of the space spanned by the vectors in problem 1? Find a basis for the space spanned by these vectors.
3. Find a basis for the null space of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ -1 & 1 & -1 & 1 \\ 3 & 5 & 3 & 5 \end{pmatrix}.$$

What is the dimension of the row space of this matrix? What is the dimension of the column space? What is the dimension of the null space? What is the rank of the matrix? Find a basis for the null space.

4. Determine whether or not the vector $\begin{pmatrix} 1 & -1 & -1 & 1 \end{pmatrix}^t$ lies in the column space of the matrix A from problem 3. Show all of your computations and / or write down your reasoning completely.