

### Third Exam

Thursday, April 10, 2003

This exam is closed book. Make sure your name is on all pages. You should put your work on your own paper, and you may keep this exam sheet upon leaving. Be sure to check your work carefully, and to show intermediate computations in a logical presentation. **Check your calculations carefully.** Each entire problem is worth 20 points

1. Write down the matrix corresponding to a linear transformation that transforms the unit square  $0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1$  to the following parallelogram.

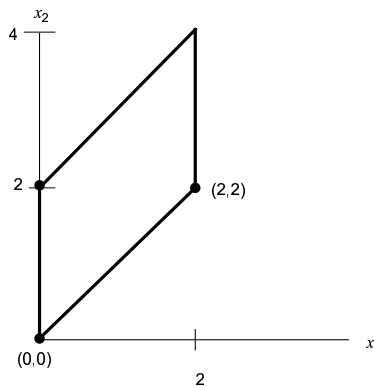


Figure 1: The image for problem 1

2. Write down the matrix for the transformation of  $\mathbb{R}^2$  that first stretches the  $x_1$  axis by a factor of 3 and the  $x_2$  axis by a factor of 2, then rotates the figure by a counterclockwise angle of  $\pi/4$ .
3. Write down the matrix for the transformation of  $\mathbb{R}^2$  that first rotates the figure counterclockwise by an angle of  $\pi/4$ , then stretches the  $x_1$  axis by a factor of 3 and the  $x_2$  axis by a factor of 2.
4. Compute the **LU** factorization of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 2 & 2 & 5 \end{bmatrix}.$$

(That is, find  $L$  and  $U$  with  $A = LU$ .)

5. Use the **LU** factorization of Problem 4 to compute the solution to  $Ax = b$ , where

$$b = [1 \quad 2 \quad 3]^T.$$