Math. 362-03
Spring, 2019
R. B. Kearfott

First Exam

Thursday, February 21, 2019

This exam is closed book, but you may use calculators that do not have computer algebra systems. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner: You will be graded on what you show, in addition to your answer. Check your work carefully. Each entire problem is worth 33 points, and 1 point is free.

1. Consider the following system of equations.

$$\begin{array}{rcrcrcl} x_1 & - & x_2 & + & x_3 & = & 1 \\ x_1 & + & x_2 & + & 2x_3 & = & 1 \\ & & 2x_2 & + & x_3 & = & 0 \end{array}$$

   (a) Write down the augmented matrix for this system of equations.

   (b) Use Gauss–Jordan elimination (Gaussian elimination with back substitution) to transform the augmented matrix to reduced row echelon form.

   (c) Use the reduced row echelon form to write down the solution set to the system of equations.

      i. If the system has no solutions, say so.

      ii. If the system has a unique solution, give the values of x_1, x_2, and x_3.

      iii. If the system has an infinite number of solutions, write the solution set in parametric form.

2. If $A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $A^{-1} = \begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix}$,
compute the solution to

$$\begin{array}{rcrcrcl} x_1 & + & \frac{1}{2}x_2 & + & \frac{1}{3}x_3 & = & 1 \\ \frac{1}{2}x_1 & + & \frac{1}{3}x_2 & + & \frac{1}{4}x_3 & = & 2 \\ \frac{1}{3}x_1 & + & \frac{1}{4}x_2 & + & \frac{1}{5}x_3 & = & 3 \end{array}$$

by hand, by doing a single matrix multiplication. Show your work.

3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$, and $v = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$,
compute the following.

   (a) $AB - BA$.

   (b) $A^2 + 2A + I$.

   (c) $v^T v$.

   (d) vv^T.