

(1) (a) Since the matrix is triangular, its determinant is:

$$(1)(2)(3)(4)(5)(6)(7) = \boxed{5040}$$

(b) $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 4 \\ 2 & 0 & 5 \end{vmatrix} = -2 \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} = -2(5-8) = (-2)(-3) = \boxed{6}$

(c) Since the second row is 2 times the first row, the determinant is $\boxed{0}$.

(2) $\text{proj}_{\vec{a}} \vec{u} = \frac{1}{\|\vec{a}\|^2} (\vec{u} \cdot \vec{a}) \vec{a} = \frac{1}{4} (2-1+2-1) (1, -1, 1, -1)$
 $= \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right)$

(3) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \text{ giving } \begin{matrix} x = z \\ y = -2z \\ z = z \end{matrix}$$

We identify z with a parameter t and $\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. We then have $\boxed{\vec{r} = t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}}$.

(4) Displacements in the plane are: $(-1, 0, 0) - (1, 0, 0) = (-2, 0, 0)$
 and $(1, 1, 1) - (1, 0, 0) = (0, 1, 1)$

A normal to the plane is thus:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix} = (-2)(-1) \begin{vmatrix} \vec{j} & \vec{k} \\ 1 & 1 \end{vmatrix} = 2(\vec{j} - \vec{k}) = (0, 2, -2)$$

For simplicity, we may take $\vec{n} = \frac{1}{2}(0, 2, -2) = (0, 1, -1)$.

An equation for the plane is thus:

$$[(x, y, z) - (-1, 0, 0)] \cdot (0, 1, -1) = 0, \text{ that is:}$$

$$\boxed{y - z = 0}$$