1 (i) Since the matrix is already in RREF, a basis for the row space is $\{[0,1,0,-1,2], [0,0,1,1,-3]\}$ and a basis for the column space is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$.

For the null space, we have:

$x_1 = x_1$
$x_2 = x_4 - 2x_5$
$x_3 = -x_4 + 3x_5$, with free variables $x_1, x_4,$ and x_5.
$x_4 = x_4$
$x_5 = x_5$

We thus have
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + v \begin{bmatrix} 0 \\ -2 \\ 3 \\ 0 \\ 1 \end{bmatrix},$$

so a basis for the null space is:
$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

(ii)
$$\begin{bmatrix} 0 & 1 & 2 & 1 & -4 \\ 0 & -1 & 1 & 2 & -5 \\ 0 & 1 & 1 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 2 & 1 & -4 \\ 0 & 0 & 3 & 3 & -9 \\ 0 & 0 & -1 & -1 & 3 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 2 & 1 & -4 \\ 0 & 0 & 1 & 1 & -3 \\ 0 & 0 & -1 & -1 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & 1 & 2 & 1 & -4 \\ 0 & 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Since RREF$(B) = A$, the row space and null space of B are the same as those of A. A basis for the column space of B is:
$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

2. Since the second row is 2 times the first row, we immediately see rank$(A)=1$. Since A has 4 columns, we also see nullity$(A) = 4-1=3$.
The dimension of the row space and the dimension of the column space are both equal to 1 (the rank).

3. $\det(A-\lambda I) = \begin{vmatrix} 2-\lambda & 0 & -1 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (2-\lambda)(1-\lambda)^2 = 0$ for an eigenvalue λ.

Therefore, ~~a basis for the null space~~ the eigenvalues are $\lambda_1 = 2$, $\lambda_2 = 1$.

~~Correspon~~ $A-\lambda_1 I = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$,

with null space $t\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Thus

$\left\{1, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}$ is an eigenvalue/eigenvector pair.

$A-\lambda_2 I = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ with null space

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + u\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. A basis for the

eigenspace corresponding to $\lambda_2 = 1$ is thus:

$\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\}$