

$$\textcircled{1} \begin{bmatrix} 1 & 1 & 1 & 1 & : & 3 \\ 1 & -1 & 1 & -1 & : & 3 \\ 2 & 0 & 2 & 0 & : & 6 \end{bmatrix}$$

$$\textcircled{2} \begin{bmatrix} 1 & 1 & 1 & 1 & : & 3 \\ 1 & -1 & 1 & -1 & : & 3 \\ 2 & 0 & 2 & 0 & : & 6 \end{bmatrix} \quad \begin{array}{l} R_2 \leftarrow R_2 - R_1 \\ R_3 \leftarrow R_3 - 2R_1 \end{array} \quad \begin{bmatrix} 1 & 1 & 1 & 1 & : & 3 \\ 0 & -2 & 0 & -2 & : & 0 \\ 0 & -2 & 0 & -2 & : & 0 \end{bmatrix}$$

$$\underline{R_3 \leftarrow R_3 - R_2} \quad \begin{bmatrix} 1 & 1 & 1 & 1 & : & 3 \\ 0 & -2 & 0 & -2 & : & 0 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix} \quad \underline{R_2 \leftarrow R_2 / (-2)} \quad \begin{bmatrix} 1 & 1 & 1 & 1 & : & 3 \\ 0 & 1 & 0 & 1 & : & 0 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$\underline{R_1 \leftarrow R_1 - R_2} \quad \begin{bmatrix} 1 & 0 & 1 & 0 & : & 3 \\ 0 & 1 & 0 & 1 & : & 0 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$\textcircled{3}$  The rank is 2, since there are only two linearly independent rows.

$\textcircled{4}$  Translating  $x_3$  and  $x_4$  to the right sides of the equations, we obtain

$$\begin{array}{l} x_1 = 3 - x_3 \\ x_2 = -x_4 \\ x_3 = x_3 \\ x_4 = x_4 \end{array} \quad \text{i.e.} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$\textcircled{5}$  The solution represents a plane in 4-dimensional space.

$$\textcircled{6} A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 2 & 0 & 2 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}.$$