Math. 362-01
Summer, 2017
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Third Exam

Friday, July 21, 2017

This exam is closed book, but you may use calculators that do not have computer algebra systems. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner: You will be graded on what you show, in addition to your answer. Check your work carefully. Each entire problem is worth 33 points, and 1 point is free.

1. Consider the following two bases for $\mathbb{R}^2$:

$$B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\} \quad \text{and} \quad B' = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}.$$

   Compute the transition matrix $P_{B \to B'}$, such that the representation of a vector in terms of the vectors in B' is $P_{B \to B'}$ times the representation in terms of B.

2. Consider:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 3 & 2 \end{bmatrix} \tag{1}$$

   (a) Compute a basis for the row space of A.
   (b) Compute a basis for the column space of A.
   (c) Compute a basis for the null space of A.
   (d) What is the dimension of the row space of A?
   (e) What is the dimension of the column space of A?
   (f) What is the dimension of the null space of A?
   (g) What is the rank of A?
   (h) What is the nullity of A?
   (i) Are the rows of A linearly independent?
   (j) Are the columns of A linearly independent?

3. Write down a single matrix that does the following, in the following order, to vectors $[x_1, x_2, x_3]^T \in \mathbb{R}^3$:

   (a) Stretches by a factor of 2 in the direction of the x_1-axis.
   (b) Contracts, with a multiplying factor of $\frac{1}{3}$ in the direction of the x_3-axis.
   (c) Rotates about the x_2-axis from the x_1-axis towards the x_3-axis, through an angle of $\pi/6$ radians.