1 (a) $V-2W = \begin{bmatrix}2\\3\\2\end{bmatrix} - \begin{bmatrix}2\\2\\2\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$

(b) $\|u-v\| = \left\|\begin{matrix}-3\\-3\\-1\end{matrix}\right\| = \sqrt{3^2+3^2+1^2} = \sqrt{19}$

(c) $\|3u-3v\| = 3\|u-v\| = 3\sqrt{19}$

(d) $u\circ(5v) = 5(u\circ v) = 5[-1,0,1]\begin{bmatrix}2\\3\\2\end{bmatrix} = 5(-2+2)=0$

(e) $(5u)\circ v = 5(u\circ v) = 0$

(f) $(u-v)\circ(u-v) = \|u-v\|^2 = 19$

2 (a) $\text{proj}_w u = \frac{1}{\|w\|}\left(\frac{w}{\|w\|}\circ(u)\right)$. $\|w\| = \sqrt{3}$

$= \frac{1}{w\circ w}(u\circ w)\,w = \left(\frac{1}{3}[-1,0,1]\begin{bmatrix}1\\1\\1\end{bmatrix}\right)\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}.$

(b) $\text{proj}_u w = \frac{1}{u\circ u}(w\circ u)\,u = \begin{bmatrix}0\\0\\0\end{bmatrix}$

(c) $u\circ(V\times W) = \begin{vmatrix}-1&0&1\\2&3&2\\1&1&1\end{vmatrix} = -1\begin{vmatrix}3&2\\1&1\end{vmatrix} + 1\begin{vmatrix}2&3\\1&1\end{vmatrix}$

$= -1(1)+1(-1) = \boxed{-2}$

~~(d)~~ 3 Direction vectors are $V = \begin{bmatrix}0\\1\\-1\end{bmatrix} - \begin{bmatrix}1\\0\\-1\end{bmatrix} = \begin{bmatrix}-1\\1\\2\end{bmatrix}$

and $W = \begin{bmatrix}1\\0\\1\end{bmatrix} - \begin{bmatrix}1\\0\\-1\end{bmatrix} = \begin{bmatrix}0\\0\\2\end{bmatrix}$.

(a) A normal to the plane is $V\times W = \begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\-1&1&2\\0&0&2\end{vmatrix}$

$= 2\begin{vmatrix}\vec{i}&\vec{j}\\-1&1\end{vmatrix} = 2\vec{i}+2\vec{j} = \begin{bmatrix}2\\2\\0\end{bmatrix}$, so an equation for the plane is:

$(x_1-1)\cdot 2 + (x_2-0)\cdot 2 = 0$, or $x_1 - 1 + x_2 = 0$.

$\boxed{\therefore\ x_1 + x_2 = 1}$

(3b) Parametric equations are:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + t\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} + u\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix},$$

That is, $\boxed{x_1 = 1-t,\ x_2 = t,\ x_3 = u}$.

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(4)
$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & -1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 3 & 2 & -3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & -1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -2 & -2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & -1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & 2 & -3 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Thus,
$$x_1 = -3 - 3x_3 - 2x_4$$
$$x_2 = 1 - x_4$$
$$x_3 = x_3$$
$$x_4 = x_4.$$

We thus have:

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + u\begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}}$$
