Math. 455-01
Spring, 2016
R. B. Kearfott

First Exam

Tuesday, February 16, 2016

This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each entire problem is worth 20 points.

1. Define the function `em1dx` by

$$\texttt{em1dx}(x) = \begin{cases} \dfrac{e^x - 1}{x} & \text{if } x \neq 0, \\ 1 & \text{if } x = 0. \end{cases}$$

   Find a polynomial approximation to `em1dx` with an absolute error of at most 10^{-3} over the interval $x \in [-0.1, 0.1]$. Show your work, so it can be seen that this accuracy is attained.

2. Consider $f(x) = (x-1)^{1/3}$.

   (a) Compute the condition number of f as a function of x.

   (b) If you are using a machine that carries 16 decimal digits, how many digits would you expect to be correct if you evaluated f at $1 + 10^{-6}$?

3. Consider the sequence

$$x_{k+1} = \frac{1}{2}x_k + \frac{1}{2}.$$

   (a) What is the limit x_* of this sequence?

   (b) What is the order of convergence of this sequence?

   (c) How many binary digits do you expect to gain on each iteration (e.g., 1, two, three, double)?

4. Suppose you are using a machine with base $\beta = 10$, $t = 2$ decimal digits, and $L = -16$, $U = 16$, with normalized numbers.

   (a) What are `HUGE` and `TINY` for this system?

   (b) What is the distance between 10^6 and the nearest number in the system greater than 10^6?

5. Rearrange the following expressions to reduce the effects of roundoff error when they are evaluated using floating point arithmetic.

   (a) $E_2(x) = (x^4 - 1)/(x - 1)$ near $x = 1$.

   (b) $\cos^2(x) - \sin^2(x)$ near $x = \pi/4$.