1 The Taylor polynomial for e^x is (around $x=0$)

$$1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \frac{x^{n+1}}{(n+1)!} e^c, \text{ so}$$

$$emidx(x) = 1 + \frac{x}{2!} + \cdots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{(n+1)!} e^c,$$

so approximation of emidx by its degree $n-1$ Taylor polynomial has error $\left|\frac{x^n}{(n+1)!} e^c\right| < \frac{3}{(n+1)!}(.1)^n$ for $|x| < 0.1$

We have

n	$\frac{3}{(n+1)!}(.1)^n$
1	0.15
2	5×10^{-3}
3	1.25×10^{-4}

Hence, $n = 3$ will do, and a sufficiently accurate polynomial is:

$$\boxed{P_2(x) = 1 + \frac{x}{2} + \frac{x^2}{6}}$$

2 (a) The condition number is

$$K_f(x) = \frac{x f'(x)}{f(x)} = \frac{x\left(\frac{1}{3}(x-1)^{-2/3}\right)}{(x-1)^{1/3}} = \boxed{\frac{1}{3}\,\frac{x}{x-1}}$$

(b) $K_f(1+10^{-6}) = \frac{1}{3}\,\frac{1+10^{-6}}{10^{-6}} = \frac{1}{3}(1+10^{-6})10^6 \approx 3 \times 10^5$

We thus lose between 5 and 6 decimal digits, so we can only expect $\boxed{10}$ digits to be correct.

3 a The limit of this sequence is $x_* = 1$.

We have $|x_{k+1} - x_*| = \left|\frac{1}{2}x_k + \frac{1}{2} - 1\right| = \left|\frac{1}{2}x_k - \frac{1}{2}\right|$
$= \frac{1}{2}|x_k - x_*|$.

The order of convergence is thus $\boxed{1}$, that is, the convergence is linear, with convergence factor $\frac{1}{2}$.

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b Since the error is decreased by a factor of 2 each iteration, we expect 1 binary digit to be gained each iteration.

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4 a HUGE $= .99 \times 10^{16}$, and TINY $= .10 \times 10^{-16}$

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b The next number is $.11 \times 10^{7}$, so the distance is
$|.11 \times 10^{7} - .10 \times 10^{7}| = |.01 \times 10^{7}| = 10^{5} =$ ~~10,000~~ $\boxed{100{,}000}$

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5 a $\frac{x^4 - 1}{x - 1} = \boxed{x^3 + x^2 + x + 1}$.

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b ~~$\sin^2(x) - \cos^2$~~ $\cos^2 x - \sin^2(x) = \boxed{\cos(2x)}$