Math. 455-01
Spring, 2005
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First Examination

Thursday, February 24, 2005

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully.

1. (20 points) Find a polynomial that approximates

$$f(x) = \left\{ \begin{array}{cc} \frac{\sin(x)}{x} & x \neq 0 \\ 1 & \text{otherwise.} \end{array} \right\}$$

to within an absolute error of 10^{-5} on the interval $x \in [-0.1, 0.1]$.

2. (40 points) Consider:

$$\frac{f(x+h) - f(x)}{h}, \qquad (1)$$

where f is as in problem 1.

   (a) Compute the quantity using (i) three significant digit rounded decimal arithmetic; (ii) all the digits in your calculator. In the first case, recall that the number of significant digits is the number of leading non-zero digits. Also, round $f(x+h)$ and $f(x)$ properly before subtracting.

   (b) Assuming the representation computed from your calculator is exact, compute the absolute and relative error in the above calculation.

   (c) Using $f(0) = 1$ and $f([0.1] \in [0.998, 0.999]$, use interval arithmetic to obtain a lower bound and an upper bound (i.e. an enclosure) of the exact quotient.

3. (20 points) Answer:

$$|\cos(x) - 1| = \mathcal{O}\,(?) \qquad \text{as } x \to 0.$$

Show your computations.

4. (20 points) Use Newton's method to compute an approximation to the smallest positive x such that $x = \cos(x)$. Iterate until the first four digits do not change. (*Hint: You may wish to graph the two functions to obtain an idea of what the initial guess should be.*)