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~~Note~~ **Note**: This is not the only set of answers: There are several ways of approaching those problems.

1. Writing down the Taylor polynomial for sin(x) and dividing by x, we obtain:

$$\frac{\sin(x)}{x} = \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!}\sin^{(5)}(c)}{x}, \text{ so}$$

$$\left|\frac{\sin x}{x} - \left[1 - \frac{x^2}{6}\right]\right| \le \frac{x^4}{5!}|\sin^{(5)}(c)| \le \frac{x^5}{5!} \le 8.4\times 10^{-8} \text{ for } |x| \le 0.1.$$

Thus, $p(x) = 1 - \frac{x^2}{6}$ is an appropriate polynomial.

2. ~~We will write~~ ~~Sin~~ $f(h) \approx .998$, so

(a) $$\frac{f(h)-f(0)}{h} \approx \frac{.998-1}{.1} \approx \boxed{\cancel{-.0167}}\ \boxed{\cancel{-.002}}\ \boxed{-.02}$$

(b) The "exact" answer is $-.016658\ldots$
So the relative error is $\frac{|-.02 + .016658\ldots|}{-.016658\ldots} \approx 0.2$.

(c) $$\frac{[0.998, 0.999] - [1,1]}{[0.1]} = \frac{[-.002, -.001]}{0.1} = [-.02, -.01].$$

This interval contains the exact result.

(3) $\cos(x) = 1 - \frac{x^2}{2}\cos(c)$, so

$|\cos(x) - 1| \le \frac{x^2}{2}$ as $x \to 0$, so

$|\cos(x) - 1| = O(x^2)$ as $x \to 0$.

(4) Set $f(x) = x - \cos(x)$, so $f'(x) = 1 + \sin(x)$.

Thus, Newton iteration is: $x \leftarrow x - \dfrac{x - \cos(x)}{1 + \sin(x)}$

The denominator is nonzero for x between 0 and $3\pi/2$. Let's start at $x = 1$. We obtain:

#	x	$f(x)$	$f(x)/f'(x)$
0	1.	.459697694	.249636132
1	.750363868	.018923074	.011250977
2	.739112891	4.645858×10^{-5}	$2.77575260 \times 10^{-5}$
3	.739085133	2.847×10^{-10}	1.701×10^{-10}
4	.739085133	0 to machine precision	