Economic Dispatch: Applying Interval-Based Dependency Analysis to an Electric Power Problem

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1 Summary

A common way to model uncertainty in the value of a quantity is to use a probability density function (PDF) or its integral, a probability distribution function (CDF). When two such values are combined to form a new value equal to their sum, product, max, etc., the new value is termed a *derived distribution*[5]. It is well-known that derived distributions may be obtained by numerical convolution, Monte Carlo simulation, and analytically for specific classes of input distributions, under the assumption that the input distributions are independent. It is also possible to obtain derived distributions for specified dependency relationships other than independence. However, it is not always the case that the dependency relationship is known. Thus there is a need for obtaining solutions without assuming independence or any other specific dependency relationship. There are two numerical algorithms that have been implemented in software for this. Numerical approaches have the advantage of applicability to a very wide class of distributions. Probabilistic Arithmetic [6] is implemented in the commercially available software tool RiskCalc [3]. Interval-Based Dependency Analysis (IBDA) [2], which extends our previous tool [1] by eliminating the independence assumption, is implemented in the software tool Statool and is available upon request from the authors. While the two tools have fundamental similarities [4], a significant difference with respect to the present problem is that IBDA supports, and Statool implements, excess width removal in the underlying interval calculations, from some expressions. In this paper we apply IBDA to generalize a solution to the well-known *economic dispatch* problem in electric power generation to the case where the dependency relationship between the fuel costs of two generators is unspecified.

2 The Problem

The economic dispatch problem in electric power generation may be stated as follows. It is desired to determine how much power should be generated by each of two generators to meet a given level of demand so that total generation cost is minimized. One of a number of approaches to solving this problem is termed LaGrangian Relaxation [7]. We incorporate uncertainty into the LaGrangian Relaxation technique for solving the sample problem by modeling uncertainty in the cost of fuel to run the generators with probability distributions, postulating in addition that the dependency between the two fuel costs of the two generators is unknown (as would occur if one generator burns oil and the other coal). The uncertainties are then propagated through the algebraic expression derived by the LaGrangian Relaxation technique.

First, we specify the cost equations as

$$F_1 = v_1(8P_1 + 0.024P_1^2 + 80), \quad F_2 = v_2(6P_2 + 0.04P_2^2 + 120),$$

where P_1 and P_2 are the power outputs of generators 1 and 2 in megawatts; v_1 and v_2 are the fuel costs for generators 1 and 2 in \$ per M Btu; and F_1 and F_2 are the generation costs for given power output levels and fuel cost rates. Therefore generation costs change nonlinearly with power output according to the following equations.

$$\frac{dF_1}{dP_1} = v_1(8 + 0.048P_1), \quad \frac{dF_2}{dP_2} = v_2(6 + 0.08P_2). \tag{1}$$

Solving the problem requires minimizing an objective function

$$F = F_1 + F_2 = v_1(8P_1 + 0.024P_1^2 + 80) + v_2(6P_2 + 0.04P_2^2 + 120),$$

subject to the constraint $P = P_1 + P_2$ where P is the total customer demand for electric power which for this example we take as 400 megawatts. This gives a constraint function

$$P = P_1 + P_2 = 400. \tag{2}$$

By the method of Lagrangian multipliers from calculus, at an extreme value of this objective function,

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \lambda \tag{3}$$

for some λ. This is derived from the Lagrange function L which relates objective function F and constraint (1) according to $L = F + \lambda \cdot P$, which implies

$$\frac{\partial L}{\partial P_1} = \frac{dF_1(P_1)}{dP_1} - \lambda = 0$$

for generator 1 and similarly for generator 2.

From (1) and (3),

$$v_1(8 + 0.048P_1) = \lambda = v_2(6 + 0.08P_2), \quad P_2 = 400 - P_1,$$

and solving simultaneous equations for P_1 gives

$$P_1 = \frac{38v_2 - 8v_1}{0.08v_2 + 0.048v_1}, \quad P_2 = 400 - P_1, \tag{4}$$

as the most economical amounts of power to generate from generators 1 and 2 to meet the demand (assuming those amounts are within the capacity of both generators). P_1 and P_2 are easily calculated for real values of v_1 and v_2, but given distribution functions for v_1 and v_2, the problem requires evaluating an expression on random variables v_1 and v_2 involving a sum, difference and quotient. Solving it by dividing a difference of random variables by a sum results in excessively wide envelopes on the CDFs for P_1 and P_2 because the same operands occur in both terms, leading to excess width in the underlying interval calculations. Instead the entire expression must be treated as a single binary operation on v_1 and v_2. Figure 1 shows the results given PDFs describing v_1 and v_2.

3 Discussion and Conclusion

Statool currently has certain limitations. Planned extensions include the following.

1. Asymptotic pdf tails. The process of discretizing a pdf into a histogram does not presently allow for the case where a pdf tail trails off to plus or minus infinity. Yet this implies setting definite bounds, though any specific such bounds might be hard to justify. Indeed unusual and extreme values can occur in the electric power domain, as happened for example in the California power crises recently. The solution is to allow the discretization to include open intervals with an end point at ∞ or $-\infty$. This in turn would require the arithmetic operations to be defined on such intervals. Fortunately this is possible, e.g., $[1, \infty) + [1, 2] = [2, \infty), (-\infty, -1] * [-2, -1] = [1, \infty), [1, 2]/[-1, 1] = (-\infty, \infty)$, etc.

2. Partial dependency. While the system currently can calculate either under the assumption of independence, or with no assumption about dependency, partial information about dependency is often present in real problems. Correlation values are a typical example. An example would be prices of different fuels, for which one would expect a generally positive correlation.

In the full paper we will explain the IBDA algorithm, and also include explanations and figures, showing how assuming independence results in stronger results, while excess width in interval evaluation of equation (4) leads to weaker results. We will also remark on the implications of the CDF bounds to decision-makers.

Figure 1: Solution for P_1 of equation (4), given the histogram-discretized PDFs for v_1 and v_2 shown. The CDF for optimum power generation from generator 1 will be within the envelopes shown regardless of the dependency

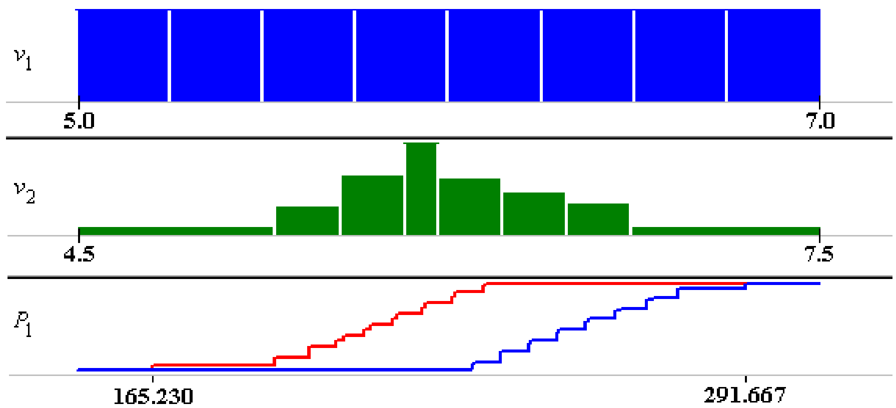

Figure 1:

relationship between inputs v_1 and v_2. The envelopes might be sufficient for a decision, or might point out the need for additional information gathering to sharpen the input distributions and/or identify their dependency relationship sufficiently to support a decision.

References

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