Validated Integration of Asteroid Orbits

Martin Berz[1] and Kyoko Makino[2]

[1]Department of Physics and Astronomy, and
National Superconducting Cyclotron Laboratory
Michigan State University
East Lansing, MI 48824, USA, berz@msu.edu
[2]Department of Physics,
University of Illinois at Urbana-Champaign,
Urbane, IL 61801-3080, USA, makino@uiuc.edu

Dedicated to Ramon Moore, who started it all

From the earliest days of self-validating methods, the integration of orbits in the solar system has ranked high on the interest of researchers; in fact the early work of Ramon Moore in this area presented one of the first practical applications of self-validated and interval methods. In this talk we focus on recent developments in this direction, specifically the integration of near-earth asteroids and the exclusion of collisions with earth. We review both previous work and show various new improvements to the performance of the technique.

The approach is based on the Taylor model approach, which allows for a description of functional dependencies without some of the limitations of direct interval methods. We review both the Taylor model approach as well as the basic ideas of the verified integrator VI. First, the method has a sharpness that scales with a high order of the domain size. Furthermore, it largely avoids the so-called dependency problem that arises in the validated evaluation of extended calculations. This is done by explictly describing dependencies on initial conditions as a high-order Taylor polynomial and a small interval remainder at any time step. Various practical examples related to this question are given. Finally, the method scales very favorably to higher dimensions and can alleviate the so-called dimensional curse. We present a few examples of the methods for validated optimization and quadrature. We also discuss the various platforms through which the methods can be used, including the C++, F90, and COSY interfaces.

When applied to the verified integration of ODEs, the method successfully avoids the so-called wrapping effect problem, which is in essence a manifestation of the dependency problem that unavoidably occurs in the very extended functional dependence of final conditions on initial conditions as prescribed by the step-by-step integration scheme. It also allows a simple combination of the frequently separated steps of finding an a priori inclusion and then subsequently performing a more sophisticated actual step. Many of the details of the methods are discussed in a talk by Kyoko Makino at this meeting.

For the purpose of integration of asteroid orbits, a far-reaching control of the wrapping effect is crucial because of the unavoidably rather large initial domains to which the dynamical quantities are known. We discuss in detail the performance of the Taylor-model based integrators for a full model of dynamics in the solar system, including relativistic and other corrections that leads to an accuracy within the kilometer domain for typical integrations. The validated methods allow a control of the overestimation to a few percent over integration periods of up to 100 years, which is sufficient for detailed study of possible collisions of near-earth asteroids with earth.