Extended Interval Power Function

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The general power function,

$$\hat{} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

$$(x, y) \mapsto \mathrm{x\hat{}y} := x^y$$

is not defined for $x < 0$.

On the other hand, well-known formulas exist for $y \in \mathbb{Z}$ or for some $y \in \mathbb{Q}$.

In the framework of an extended interval arithmetic computing containment sets for every function these values should be included in the range, if the domain is accordingly extended.

$$[-2, 2]^{[0.9,1.1]} \supseteq [-2, 2]^{[1.0,1.0]} \supseteq [-2, 2]$$

One goal of containment arithmetic is to provide an exception free evaluation of functions over an arbitrary range. In current libraries like Sun's [1] or ours [2] the power function is defined for positive radicands only. As a consequence the result with the given sample values is

$$[-2, 2]^{[0.9,1.1]} = [0, 2]^{[0.9,1.1]} = [0, 2^{1.1}] = [0, 2.14]$$

In this paper we discuss alternative implementations of the power function, compare them with computer algebra systems, develop containment sets and discuss the issue of accuracy.

References

[1] Sun Microsystems, `C++` *Interval Arithmetic Programming Reference*, October 2000
`http://www.sun.com/forte/cplusplus/interval/index.html`

[2] W. Hofschuster et al., *The Interval Library* `fi_lib++ 2.0`, *Design, Features and Sample Programs*, Preprint 2001/4, Universität Wuppertal, Dec. 2001
`http://www.math.uni-wuppertal.de/wrswt/literatur.html`

[3] G. W. Walster, E. R. Hanson, and J. D. Pryce, *Extended Real Intervals and the Topological Closure of Extended Real Numbers*, June 1999.