A Numerical Study on a New Heuristical Decision Index for Interval Global Optimization

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The talk gives an overview on the numerical test results of solving inequality constrained global optimization test problems with interval Branch-and-Bound methods.

In [1, 3] a new heuristic decision index was discussed for *unconstrained* problems and investigated in detail. This index has the form of $p\hat{f}(\boldsymbol{X}) := (\hat{f} - \underline{\boldsymbol{f}}(\boldsymbol{X}))/w(\boldsymbol{f}(\boldsymbol{X}))$, where $\boldsymbol{X}$ is an interval vector, $\hat{f}$ is an approximation of the global minimum value and $\boldsymbol{f}$ denotes the interval inclusion function of the objective function. This index measures the relative position of the minimum within the inclusion $\boldsymbol{f}(\boldsymbol{X})$ and it is suitable to be applied as a subinterval selection criterion and as a part of the subdivision rule as a decision factor.

J. F. Hernández proposed the idea of extending this index for constrained problems by taking the effect of the constraints into account in a similar way:

$$pu\boldsymbol{g}_j(\boldsymbol{X}) := \min\left\{\frac{-\underline{\boldsymbol{g}_j}(\boldsymbol{X})}{w(\boldsymbol{g}_j(\boldsymbol{X}))}, 1\right\}, \quad pu(\boldsymbol{X}) := \prod_{j=1}^{r} pu\boldsymbol{g}_j(\boldsymbol{X}).$$

(where $\boldsymbol{g}_j$ is the interval inclusion function of the jth constraint). The pu quantity measures the relative position of 0 within the inclusions of the constraint functions, i.e. the feasibility of the box $\boldsymbol{X}$. Finally, the heuristical decision index for constrained problems is formalized by $pup(\hat{f}, \boldsymbol{X}) := pu(\boldsymbol{X}) \cdot p(\hat{f}, \boldsymbol{X})$. We can conclude that if the pup value for a given box is high, then the box should be preferred for an early selection (interval selection step), or it is advisable to split it into a higher number of subboxes (subdivision step).

In the numerical tests we were dealing with two different types of problems: the first was the problem class of the obnoxious facility location model [4].

For such a problem our goal is to place an unpleasing object into a region by considering the disappointment of the inhabitants (described by an exponential objective to be minimized) and the geographical possibilities (modelled by linear and quadratic constraint functions).

The second part of the test problems came basically from unconstrained global optimization; we have selected some harder problems, e.g. Hartman-6, Goldstein-Price, Levy-3, Ratz-4 and EX2 (for the definitions see [2, 6, 7]) and completed them with sets of randomly generated linear and quadratic constraints.

The main consequence of our investigations is that the new type of interval selection criterion significantly improves the efficiency in terms of both the running time and the memory complexity. In addition, the largest improvements were achieved on the hardest problem instances. We found that it is worth to make further investigations of our methods on other type of hard constrained problems.

Acknowledgements

This work was supported by the Grants FKFP 0449/99, OTKA T 32118 and T 34350.

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