Generation of Bode and Nyquist Plots for Nonrational Transfer Functions to Prescribed Accuracy

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Abstract

We present interval analysis based procedures for construction of the well-known Bode and Nyquist frequency response plots for nonrational transfer functions. The proposed procedures can be used to construct the plots reliably and to a prescribed accuracy over a user-specified frequency range. The procedures overcome the limitations of the only available method for nonrational transfer functions that is based on arbitrary gridding of the given frequency range. Several important examples drawn from various branches of engineering are used to demonstrate the merits of the proposed procedures.

Keywords: Frequency Response, Interval Analysis, Bode plots, Nyquist plots, Nonrational Transfer Functions.

1 Introduction

For over five decades, the Bode and Nyquist frequency response plots have been of great use in frequency domain analysis and synthesis of linear systems, see, for instance, [1, 6, 11]. For transfer functions (TFs) having a rational form, an automatic frequency grid selection procedure is available in the MATLAB toolbox [4] to generate the frequency response plots. However, this procedure has several limitations:

1. it does not guarantee that the generated plots are of a user-specified accuracy,

2. it uses an unreliable phase unwrapping procedure that can be fooled if a suitably fine frequency grid is not chosen in the frequency region having sharp phase changes, and

3. it is not applicable to the large and important class of nonrational transfer functions.

The class of nonrational transfer functions is of great practical importance, especially in chemical process control where virtually every process has significant time-delays (a time delay is modeled as a $e^{-\tau_d s}$ term, where τ_d is the amount of time delay, and this leads to a nonrational transfer function). Some of the application areas where nonrational transfer functions can be found are:

1. pressure fluctuations in a long flexible hose-tube connecting servo-valve to actuator in hydraulic servo system [3],

2. feedback system with measurement time delays [10],

3. heating of a one dimension metal rod along its length by a steam chest [12],

4. heat-exchanger systems [12],

5. multi-modal reactor systems in nuclear reactors [2], and

6. flexible or smart structures.

At the present time, the *only* method for generating the Bode and Nyquist frequency response plots for such nonrational transfer functions is through arbitrary rastering or gridding of the frequency range of interest. However, as is well-known, this so-called gridding method has significant limitations: (a) the number of grid points required to obtain a specified accuracy is unknown, and (b) for a given frequency response plot, the amount of error present is unknown, i.e., no error estimates are available. These limitations show up particularly severely when the frequency responses exhibit single or multiple sharp peaks or dips (this happens for the application systems we mentioned above). Despite the severe limitations of the gridding method, surprisingly little effort has been made in the literature to overcome them.

In this work, we propose a procedure each to generate the Bode and Nyquist frequency response plots for nonrational transfer functions. Since our procedures are based on a Vector - Adaptive subdivision and evaluation strategy, we call them as *VA* procedures. VA procedures are guaranteed to automatically generate the plots *reliably* and to a *prescribed* accuracy, throughout a given frequency range. The VA procedures are applicable to a very general class of transfer functions in the continuous as well as in the discrete-time domains. Transfer functions involving a composition of time-delay and transcendental terms can be handled equally easily in the VA procedures, without the need for any approximations. Moreover, error estimates are readily available from all plots that have been generated by the VA procedures.

2 A Procedure for Bode Plot Generation

We present the proposed VA procedure for Bode plot generation. A similar procedure can be given for the Nyquist plot generation.

The Vector-Adaptive Procedure (VA) for Bode Plot Construction

- Inputs : An expression for the transfer function $g(s)$, the frequency interval Ω of interest, and the specified maximum width ε of each magnitude and phase rectangle in the generated Bode plot. In general, ε can be different for magnitude and phase plots.
- Output: A collection of magnitude and phase rectangles, each of width at most ε, and enclosing the actual Bode magnitude and phase plot.

BEGIN Procedure

1. From the transfer function expression $g(s)$, obtain the magnitude and phase expressions $f_{mag}(\omega)$ and $f_{phase}(\omega)$, where ω is the frequency variable.
2. Construct natural interval extensions $F_{mag}(\Omega), F_{phase}(\Omega)$ for $f_{mag}(\omega)$, $f_{phase}(\omega)$, respectively.
3. Set current frequency subinterval as Ω and set the solution list L^{sol} as empty.
4. (Adaptive subdivision and vectorized evaluation)
   (a) Subdivide all current frequency subintervals, and discard the original subintervals.
   (b) Using vectorized operations, perform vectorized *evaluation* of $F_{mag}(\Omega)$ over the frequency subintervals obtained in above substep.
   (c) Deposit all magnitude rectangles whose widths are less than ε in the solution list L^{sol}, and discard the corresponding frequency subintervals from further processing [1]. Keep the remaining frequency subintervals in the current frequency list for further processing.
   (d) If there are no more frequency subintervals left for processing, go to the following step. Else, go back to the beginning of this step (of adaptive subdivision and vectorized evaluation), and repeat.
5. Output the generated Bode magnitude plot as the collection of all magnitude rectangles present in the solution list L^{sol}.
6. Repeat the above three steps but for $F_{phase}(\Omega)$. Output the generated Bode phase plot as the collection of all phase rectangles present in the solution list L^{sol}.

END Procedure**.**

[1]The corresponding frequency subintervals are no longer needed as these have produced small enough magnitude rectangles which have been just stored.

3 Results and Discussion

We test the performance of the proposed VA procedures on several real-life nonrational transfer function examples. We also test them on some challenging rational transfer function examples. The examples are chosen from the application problems listed above.

We program the VA procedures using the interval analysis toolbox INTLAB [14] in the MATLAB environment. We carry out all computations on a PC/Pentium-III 550 MHz machine. In all the examples, we set the prescribed accuracy as $\varepsilon_{\text{mag}} = 1$ decibel (dB) and $\varepsilon_{\text{phase}} = 1\,\text{deg}$. This means that magnitude (resp. phase) side of each box in the plot is to have a width at most of 1 dB (resp. 1 deg.).

We compare the frequency response plots generated using the VA procedures with those obtained using conventional rastering or gridding of the frequency interval, for three different grids of $10^2, 10^3$, and 10^4 grid points. Further, we benchmark all results against the plot obtained using a very dense grid of 5×10^5 grid points.

The results of the examples show that the gridding method yields large errors in the plots, if the frequency grid size is not carefully chosen. For instance, in our examples, it was found that grids of 10^4 grid points were often required, and in some extreme cases, even grids of 10^5 grid points were inadequate to obtain the same accuracy. Further, the accuracy of the obtained frequency response plots is unknown unless and until these are benchmarked against the "exact" plots (hopefully obtained using very dense grids). Without a good estimate of the grid points to be used and of the error present in the generated plots, there is every *danger* that one may be lead to erroneous analysis and synthesis results. The proposed procedures relieve the user of the difficulties associated with grid point selection and lack of error estimates.

(The Table containing the comparative analysis of errors, and the plots of frequency responses, are not given here due to space constraints but will be given in the full paper).

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