Kite: A New Inclusion Function for Optimization

Tamás Vinkó[1], Jean-Louis Lagouanelle[2], and Tibor Csendes[1]

[1]University of Szeged
Institute of Informatics, Szeged, Hungary
emails {tvinko,csendes}@inf.u-szeged.hu
[2]Lab. LIMA
Institut de Recherche en Informatique de Toulouse
France, email lagouane@irit.fr

Interval global optimization algorithms based on branch-and-bound methods provide guaranteed and reliable solutions for the problem

$$\min_{x \in X} f(x),$$

where the objective function $f : D \subseteq \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable and $X \subseteq D$ is the search box representing bound constraints for x. The aim of this work is to improve the efficiency by a tighter interval inclusion function, in particular we deal with *lower bounds* of f, because the guaranteed upper bound of the global minimum values can be obtained by a single function evaluation. The quality of an enclosure method is important in the implementation of the interval global optimization algorithms, because narrower enclosure of f may provide faster convergence.

In this paper the kite inclusion function is presented for branch-and-bound type interval global optimization using at least gradient information. In the one dimensional case [3] the basic idea comes from the simultaneous usage of the centered forms and the linear boundary value forms [2]. Figure 1 shows that the graph of f is within the convex inclusion cone determined by the points $(a, f(a)), S$ and $(b, f(b))$ and outside the concave exclusion cone MPN. In this figure the lines L and U mean the lower and the upper bound of the derivative of $f(x)$, while the lower bounds for the function $f(x)$ given by the centered form, the linear boundary value form and the kite are $\underline{F}_{CF}$, $\underline{F}_{LBVF}$ and $\min\{y_R, y_T\}$, respectively.

This leads to the assertion that the simultaneous usage provides a not worse enclosure of the objective function. The best choice for the center of the kite (the point (c, P) in Figure 1) correspond to the case $y_R = y_T$. The existence and uniqueness of this case can be shown in two ways: in a geometrical and in a constructive way. The isotonicity and at least quadratical convergence hold

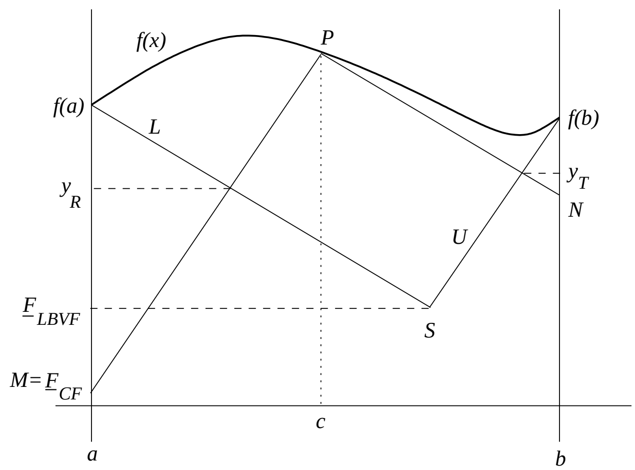

Figure 1: Simultaneous usage of the centered form (based on the middle point of the current interval) and the linear boundary value form.

and there is a pruning effect of the kite which is derived from the construction of the inclusion, thus more function evaluation is not needed to use it.

The new method can easily be implemented in a branch-and-bound type interval global optimization algorithm. For a single inclusion larger computation effort is needed by the kite algorithm, because we use the function values not only at the optimal center but at the extremal points of the examined interval.

Numerical investigations on 40 standard multiextremal test functions have been done to show the performance. For the one dimensional problems our results are summarized in Table 1, where the performance of the kite with and without its pruning effect (k+pr and k−pr, respectively) is compared to the centered form. The columns contain the number of function, derivative, and Hessian evaluation, number of bisection and the necessary list length. The percent values give the average values for the complete test set for the first and the second order algorithms.

F-eval.		D-eval.		H-eval.		bisection		list length	
k+pr	k−pr	k+pr	k−pr	k+pr	k−pr	k+pr	k−pr	k+pr	k−pr
66%	74%	39%	57%	–	–	60%	87%	56%	63%
79%	84%	64%	69%	111%	120%	76%	83%	78%	78%

Table 1: Numerical results.

In the multi dimensional case the kite inclusion function can be based on the centered forms and the method of the supporting hyperplanes [1]. Another idea is the application of the componentwise approach, where the multidimensional case can be lead back to the one dimensional case. These ideas should be investigated in the future.

Acknowledgments. This work have been supported by the grants OTKA T 034350 and OTKA T 32118.

References

[1] F. Messine and J.-L. Lagouanelle, "Enclosure Methods for Multivariate Differentiable Functions and Application to Global Optimization", *Journal of Universal Computer Science*, 1998, Vol. 4, No. 6, pp. 589–603.

[2] A. Neumaier, *Interval Methods for Systems of Equations*, Cambridge University Press, Cambridge, 1990.

[3] T. Vinkó, J.-L. Lagouanelle, and T. Csendes, *A New Inclusion Function for Optimization: Kite – The One Dimensional Case*, 2001; Submitted for publication; Available at: `http://www.inf.u-szeged.hu/~tvinko/kite.ps.gz`