Moore's Single-Use-Expression Theorem on Extended Real Intervals*

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Moore [1] proved that conditions exist when a computed interval's value is the expression's exact range. The conditions are: the expression is valid (no division by zero), rational, and real (not extended real); and each interval variable occurs no more than once in the expression. Unfortunately, in the set of extended real numbers, denoted $\mathbb{R}^* = \mathbb{R} \cup \{-\infty, +\infty\}$, Moore's single-use-expression theorem is not always true. Division asymmetry in the $\mathbb{R}^*$ number system is the root cause of the problem.

This paper further extends the $\mathbb{R}^*$ number system to remove the asymmetry from extended real division so that Moore's single-use-expression result holds. The new system is denoted $\mathbb{R}^{**}$. The new system is also applied to the complex plane to show that closed complex interval systems can be based on sets in the $\mathbb{R}^{**} \otimes \mathbb{I}^{**}$ system. Interval implementations are easily developed within the IEEE 754 floating point standard.

References

[1] R. E. Moore, *Interval Analysis*, Prentice-Hall, Englewood Cliffs, N.J., 1966.