Robust and Optimal Control of Uncertain Dynamical Systems with State-Dependent Switchings Using Interval Arithmetic*

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Abstract

In this paper, interval arithmetic techniques for controller design of nonlinear dynamical systems with uncertainties are summarized. The main reason for the application of interval techniques in this context is the quantification of the influence of uncertainties and modeling errors. They result from neglecting nonlinear phenomena in the mathematical description of real-world systems. Furthermore, measured data usually do not provide exact information about the system parameters. Often simplifications of nonlinear models or controller structures are necessary to enable the implementation of controllers. Therefore, these uncertainties have to be taken into account during the design of controllers for verification of observability, controllability, stability, and robustness.

Keywords: interval arithmetic, controller design, optimization, approximation techniques, safety-critical applications

AMS subject classifications: 65G20, 65G40, 65K10, 34H05, 49J15, 49K15, 93C15, 93D09

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1 Introduction

This paper gives an overview of general interval arithmetic procedures for the design of controllers for continuous-time dynamical systems [9]. To guarantee robustness with respect to uncertain initial conditions as well as uncertain system parameters, interval variables and verified computational techniques are taken into account during all design stages. For that purpose, the notion of optimality of control strategies is extended to systems with uncertainties by minimization of guaranteed upper bounds of suitable performance indices.

In the first part of this paper, techniques for the verification of controllability, reachability, and observability of states in the presence of uncertainties as well as stabilizability of instable systems are demonstrated. These techniques are based on the verified evaluation of the corresponding system theoretic criteria [6, 3]. Therefore, underlying computational approaches are required which yield guaranteed enclosures of all reachable states of dynamical systems. Especially for safety-critical applications, asymptotic stability and compliance with given restrictions for the state variables have to be assured for all possible operating conditions using analytical or numerical techniques. For both open-loop and closed-loop control systems, possibilities for the combination of verified techniques with classical approaches for robust and stability-based controller design in the time-domain are highlighted (Sections 2–4).

In the second part, procedures for structure and parameter optimization of continuous-time dynamical systems with bounded uncertainties are derived that rely on the above-mentioned basic concepts [8]. For that purpose, an interval-based algorithm is presented. This algorithm computes approximations of globally optimal open-loop and closed-loop control laws for dynamical systems. Focusing on continuous-time applications, we present new strategies which allow to combine piecewise constant approximations of optimal control laws with continuous approximations (Sections 5,6).

The focus of the third part is the application of the previously introduced methods and procedures to the design of robust and optimal control strategies for dynamical systems with state-dependent switching characteristics [8]. In practically relevant applications, the fact that control variables are usually bounded has to be taken into account directly during design. This aims at building a new unified, general-purpose framework for the verified synthesis of dynamical systems integrating trajectory planning, function-oriented and safety-related controller design, as well as robustness and optimality assessment and verification. Computational routines for the previously mentioned tasks already exist. However, they are usually not integrated into a common tool verifying the design using interval arithmetic (Section 6).

2 Control-Oriented Modeling of Dynamical Systems

In control engineering, commonly two tasks are distinguished: the design of open-loop control strategies and observer-based closed-loop control strategies. In the following, we restrict the discussion to continuous-time dynamical system models which are described by sets of (nonlinear) ordinary differential equations

$$\dot{x}_s(t) = f(x_s(t), p(t), u(t), t) \tag{1}$$

and nonlinear mathematical models specifying the sensor characteristics

$$y(t) = h(x_s(t), u(t), q(t), t) \quad . \tag{2}$$

In (1) and (2), the state vector is denoted by $x_s(t)$, the control vector by $u(t)$, and the parameter vectors corresponding to uncertainties and disturbances by $p(t)$ and $q(t)$, resp. Finally, the reference signal $w(t)$ describes the trajectories of the desired system outputs, where $y(t) = w(t)$ holds in the ideal case. For open-loop control strategies (Fig. 1) the control sequence $u(w)$ only depends on the reference signal $w(t)$ and (often) a finite number of its time derivatives. In contrast to the open-loop case, information on the current system states $x_s(t)$ is fed back additionally in closed-loop control structures (Fig. 2).

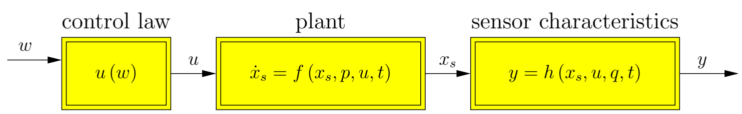

Figure 1: Open-loop control of nonlinear dynamical systems.

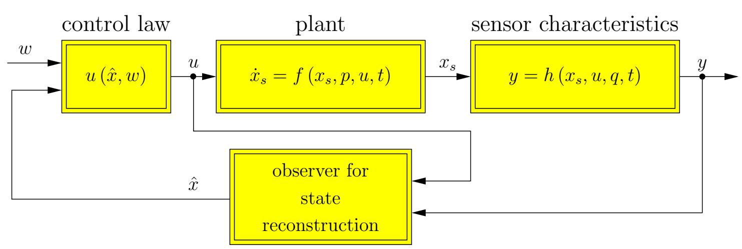

Figure 2: Observer-based closed-loop control of nonlinear dynamical systems.

If the complete state vector $x_s(t)$ and the parameters $p(t)$ and $q(t)$ are not directly accessible for measurements, estimated values $\hat{x}(t)$ are used as a substitute for these quantities in the implementation of the feedback controller. These estimates are determined with the help of state observers (cf. [6]). They are computed in terms of the known control $u(t)$ and the measured output $y(t)$.

The goal of this paper is to highlight the components of a general-purpose software tool which uses interval arithmetic routines to quantify the influence of uncertainties resulting from uncertain initial system states $x_s(0) \in [\underline{x}_{s,0} \; ; \; \overline{x}_{s,0}]$ as well as uncertain parameters $p(t) \in [\underline{p}(t) \; ; \; \overline{p}(t)]$, $q(t) \in [\underline{q}(t) \; ; \; \overline{q}(t)]$ at the earliest possible design stages. Thus, a system design becomes possible which does not rely on the implementation of controllers for a simplified, nominal model with an a-posteriori assessment of the influence of uncertainties. This often leads to tedious iterations in the design until robustness of the complete control system can be shown for all possible values of the uncertain quantities. Instead, uncertainties are propagated through the complete design process by guaranteed bounds on the system states computed by verified techniques.

The basic properties of control systems to be addressed are (global) asymptotic stability and stabilization of instable plants, improvement of the dynamics of the open-loop system, robustness and optimality, as well as feasibility of the necessary open-loop and closed-loop control laws that are required to minimize the control errors $\|w(t) - y(t)\|$ and $\|x_s(t) - \hat{x}(t)\|$, resp., for all $t \geq 0$.

3 Classical Approaches for Robust Control

In the following, an overview of the most important design strategies for robust controllers of linear and nonlinear dynamical system models is given. As discussed in this section, the use of interval techniques and other verified approaches during the complete control system design is not yet state-of-the-art in industrial control applications. Instead, approximations and simple design techniques based on linear system theoretic properties are often employed.

For linear finite-dimensional time-invariant system representations described by rational transfer functions, i.e., system models without time delay, pole placement is usually performed to parameterize linear state feedback controllers for single-input-single-output systems using Ackermann's formula (1972). This formula provides a unique solution for the feedback gains K in $u(t) = -K \cdot x_s(t)$. For multiple-input-multiple-output systems, additional constraints (such as the decoupling of the influence of input and output variables) are taken into account to find a unique solution of the feedback gain matrix K. In both cases, controllability (as introduced by Kalman in 1960) is a sufficient condition for the assignment of arbitrary poles to the closed-loop control system. Furthermore, observability is necessary if not all state variables are directly accessible by measurements such that they have to be reconstructed using model-based estimators as, for example, shown in Fig. 2.

Extensions that are commonly used for the design of controllers for systems with uncertainties and nonlinearities are:

- The Kharitonov criterion (1978) to prove asymptotic stability of characteristic polynomials with interval coefficients which is basically a generalization of the Hurwitz criterion for nominal system models.
- The parameter space approach by Ackermann and Kaesbauer [1], which is implemented in the MATLAB toolbox PARADISE based on the boundary crossing theorem by Frazer and Duncan (1929). It shows that the poles of rational transfer functions depend continuously on continuous parameter variations (Γ-stability).
- Extensions to nonlinear systems with sector-bounded static nonlinearities on the basis of the Popov criterion (1962).
- Handling of constant time delays using the notion of $\mathcal{B}$-stability.
- Design approaches based on control Lyapunov functions as well as symbolic design approaches relying on state-feedback linearization, backstepping, and flatness-based feedforward and feedback control [3, 6].

However, general approaches for controller design of nonlinear dynamical systems with bounded parameter uncertainties and bounded errors in the state reconstruction are not yet commonly used by engineers in industrial applications. Therefore, interval-based implementations of both classical design approaches and novel numerical techniques are inevitable to obtain general software tools for verification and validation of controller design.

4 An Interval-Based Framework for Robust and Optimal Controller Design

4.1 Overview

In this section, selected components of an interval arithmetic framework for verified design of robust and optimal control strategies are presented [8, 9], see Fig. 3. They are extensions of classical techniques for the analysis of asymptotic stability of dynamical systems, parameter and structure optimization based on dynamic programming, and trajectory planning techniques using information about reachability and observability of states. Especially with respect to optimal control of dynamical systems with uncertainties, non-verified approaches based on Bellman's dynamic programming and Pontryagin's maximum principle have also been used in other interval-based approaches [4, 5].

4.2 Verified Reachability Analysis

Reachability analysis for nonlinear input-affine dynamical systems

$$\dot{x}\left(t\right) = f\left(x\left(t\right)\right) + g\left(x\left(t\right)\right) \cdot u\left(t\right) \;\; , \quad x \in \mathbb{R}^{n_x} \tag{3}$$

is the prerequisite for the design of most closed-loop control strategies. To analyze the applicability of control sequences $u\left(t\right)$ such that the state variables can be influenced in a desired way, the Lie brackets of $f(x)$ and $g(x)$ have to be evaluated, which are defined according to

$$\left[f\left(x\right), g\left(x\right)\right] := \frac{\partial g(x)}{\partial x} f\left(x\right) - \frac{\partial f\left(x\right)}{\partial x} g\left(x\right) \quad . \tag{4}$$

Using (4), the state-dependent reachability matrix $P(x)$ defined by $P(x) := \begin{bmatrix} P_0\left(x\right) & P_1\left(x\right) & \dots & P_{n_x-1}\left(x\right)\end{bmatrix}$ with $P_0\left(x\right) := g\left(x\right)$, $P_1\left(x\right) := \left[f\left(x\right), g\left(x\right)\right]$, $\dots$, $P_k\left(x\right) := \left[f\left(x\right), P_{k-1}\left(x\right)\right]$, $k = 2, \dots, n_x - 1$ can be obtained.

The rank of the matrix $P(x)$ is usually state-dependent. To guarantee that all states can be influenced in a desired way, $P(x)$ must have full rank n_x along the complete trajectory for all possible values of the uncertain system parameters and all possible state variables. For linear systems, the rank criterion for the reachability matrix is identical to Kalman's criterion for state controllability.

4.3 Verified Observability Analysis

Similarly, the observability matrix $Q(x)$ is defined by

$$Q(x) := \left[\left(\frac{\partial h(x)}{\partial x} \right)^T \quad \left(\frac{\partial L_f h(x)}{\partial x} \right)^T \quad \dots \quad \left(\frac{\partial L_f^{n_x-1} h(x)}{\partial x} \right)^T \right]^T \tag{5}$$

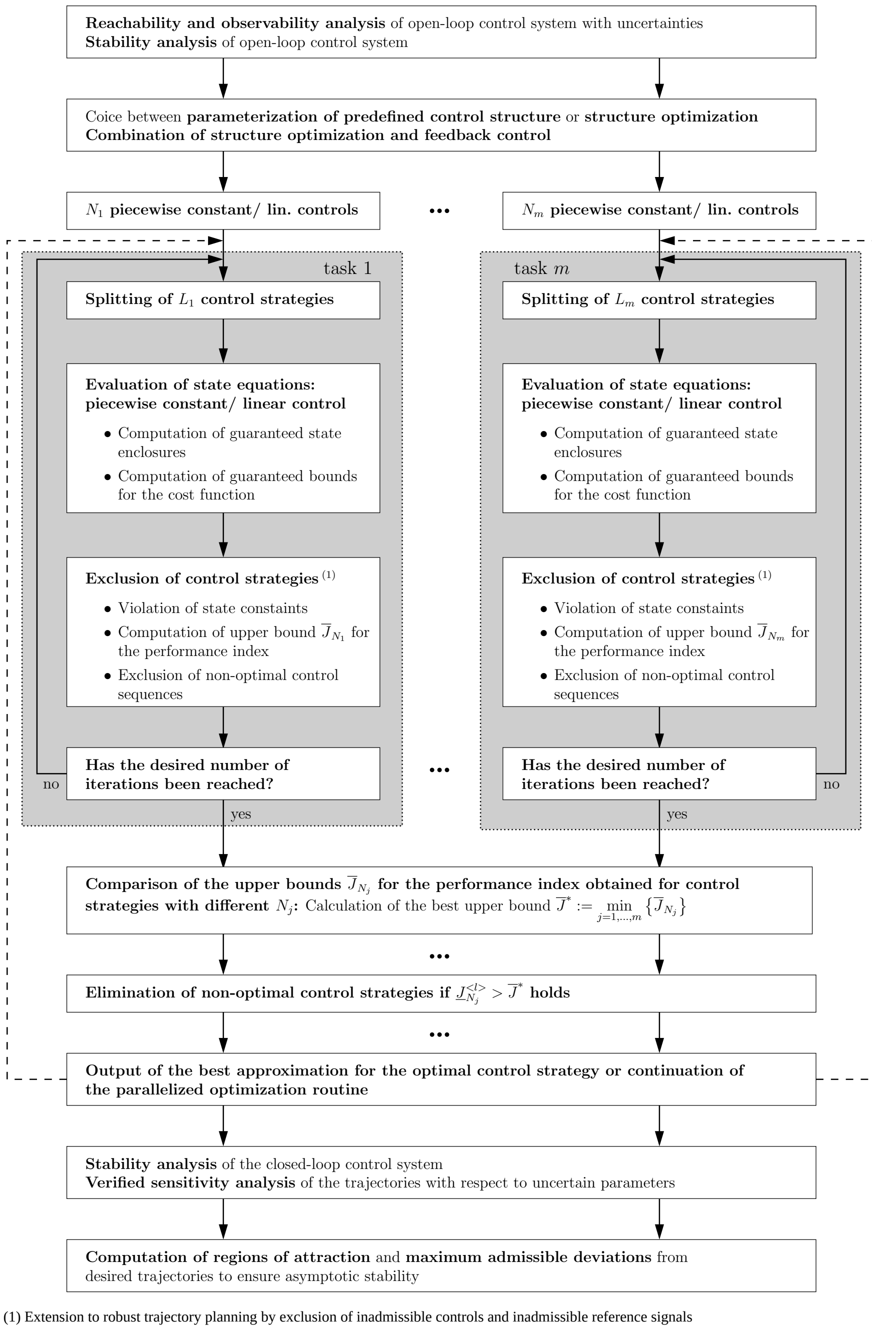

Figure 3: Components of an interval-based framework for the design of optimal and robust controllers.

with the Lie derivatives $L_f^i h(x) := L_f\left(L_f^{i-1}h(x)\right)$ with $L_f^0 h(x) := h(x)$ and $L_f h(x) := \frac{\partial}{\partial x}h(x)\cdot f(x)$. These Lie derivatives provide information about the variation of the output $y = h(x)$ of the dynamical system along the vector field $f(x)$. Similar to the reachability analysis, the rank of the matrix $Q(x)$ is usually state-dependent. For linear systems, the procedure leads to Kalman's observability matrix.

Using techniques for automatic differentiation, the matrices $P(x)$ and $Q(x)$ can also be constructed for systems with uncertainties, where enclosures of the sets of reachable states are determined by verified ODE solvers such as VNODE, VSPODE, ValEncIA-IVP, or COSY VI. A sufficient condition for the applicability of a specific procedure for controller design is that both matrices have full rank for the desired trajectories [9].

4.4 Controller Design Using Input-to-State Linearization

The use of verified computational methods is shown exemplarily for the design of nonlinear controllers aiming at input-to-state linearization. The goal is to convert the input-affine dynamical system (3) into a set of linear ODEs

$$\dot{z}(t) = A\cdot z(t) + B\cdot w(t) \quad \text{with} \quad y(t) = C\cdot z(t) \tag{6}$$

using diffeomorphism $T : D \subset \mathbb{R}^n \mapsto \mathbb{R}^n$ defining a diffeomorphism $z = \tau(x) : D \subset \mathbb{R}^{n_x} \mapsto \mathbb{R}^{n_x}$ and a control law $u = r(x) + W(x)\cdot w$.

Let δ_i be the relative degree of the output y_i, $i = 1,\ldots,m$, i.e., the smallest order of the derivative $d^{\delta_i}y/dt^{\delta_i}$ which explicitly depends on the control input u. Then, the state transformation

$$z = \tau(x) = \begin{bmatrix}\tau_1^1(x) & \ldots & \tau_1^{\delta_1}(x) & \tau_2^1(x) & \ldots\end{bmatrix}^T \tag{7}$$

can be computed using the Lie derivatives $\tau_i^{r_i} = L_f^{r_i-1}h_i(x)$, $r_i = 1,\ldots,\delta_i$.

The feedback control law $u = r(x) + W(x)\cdot w$ is given by

$$r(x) = -D^{-1}(x)\varphi(x) \quad \text{and} \quad W(x) = D^{-1}(x)\ . \tag{8}$$

In (8), the vector

$$\varphi(x) = \begin{bmatrix}\varphi_1(x) & \varphi_2(x) & \ldots & \varphi_m(x)\end{bmatrix}^T_{x(t)=\tau^{-1}(z)} \tag{9}$$

is defined by $\varphi_i(x) = L_f^{\delta_i}h_i(x)$ for all $i = 1,\ldots,m$. Furthermore, the decoupling matrix $D(x)$ is computed by

$$D(x) = \begin{bmatrix} L_{g_1}L_f^{\delta_1-1}h_1(x) & \cdots & L_{g_m}L_f^{\delta_1-1}h_1(x) \\ L_{g_1}L_f^{\delta_2-1}h_2(x) & \cdots & L_{g_m}L_f^{\delta_2-1}h_2(x) \\ \vdots & \vdots & \vdots \\ L_{g_1}L_f^{\delta_m-1}h_m(x) & \cdots & L_{g_m}L_f^{\delta_m-1}h_m(x) \end{bmatrix}\ . \tag{10}$$

Linear feedback controllers can be designed for (6) if $\text{rank}\{D(x)\} = n_x$ and $\delta = \delta_1 + \ldots + \delta_m = n_x$ hold for all desired states $x(t)$, all possible parameter values p, and uncertainties of $x(t_0)$. These prerequisites can be verified by interval evaluation of $D(x)$ for guaranteed enclosures $[x(t)]$ of the sets of all reachable states. Note that the derivatives which are required to evaluate (9) and (10) have already been computed by automatic differentiation for the reachability and observability matrices $P(x)$ and $Q(x)$.

4.5 Verified Stability Analysis

An approach for verified stability analysis which is used in MATLAB and C++ routines for controller design implemented by the authors is summarized in the following. It is based on a procedure described in [2]. After computation of the interval enclosure $[x_\infty]$ of the equilibrium of the system (1), a double-valued approximate solution $\tilde{x}_\infty \in [x_\infty]$ is chosen. Then, an approximation A of the Jacobian is determined for $\tilde{x}_\infty$. Using this matrix, the Lyapunov equation

$$A^T P + PA = -I \qquad \text{with} \qquad A := \left.\frac{\partial f}{\partial x}\right|_{x=\tilde{x}_\infty} \tag{11}$$

is solved for the symmetric matrix P. If P is positive definite, i.e., if the linearized system can be proven to be asymptotically stable, an estimate for the region of attraction of the asymptotically stable equilibrium of the original nonlinear system can be determined.

For that purpose, an interval box $[x_0]$ for which $[x_\infty] \subset [x_0]$ holds is assumed. Additionally, this box must not contain further equilibria. Typically, the initialization of the interval Newton iteration used to determine $[x_\infty]$ is chosen at this stage since it fulfills the above-mentioned requirements. To analyze the stability of the dynamical system, the quadratic Lyapunov function

$$V(x,p) = (x - x_\infty)^T \cdot P \cdot (x - x_\infty) \quad , \tag{12}$$

with P determined in (11), is used. For the time derivative of this Lyapunov function, the properties

$$\left.\dot{V}(x,p)\right|_{x=x_\infty} = 0 \qquad \text{and} \qquad \left.\frac{\partial \dot{V}(x,p)}{\partial x}\right|_{x=x_\infty} = 0 \tag{13}$$

hold. Then, the Hessian

$$H := -\frac{\partial^2 \dot{V}(x,p)}{\partial x^2} \tag{14}$$

has to be positive definite for all $x \in [x_0]$. This can be shown using a procedure described by Rohn in [11].

A symmetric interval matrix $[H] = \{H | \, H_c - \Delta \leq H \leq H_c + \Delta\}$ with $H_c = \frac{1}{2}\left(\underline{H} + \overline{H}\right)$ and $\Delta = \frac{1}{2}\left(\overline{H} - \underline{H}\right)$ is positive definite if the following $2^{n_x - 1}$ point matrices $H_z = H_{-z}$ are positive definite. The matrices H_z are defined according to $H_z := H_c - T_z \cdot \Delta \cdot T_z$ with $T_z := \mathrm{diag}(z)$. The vector z has to be replaced by all possible combinations of the components $z_i = \pm 1$, $i = 1, \ldots, n_x$. Thus, $H_z = H_{-z}$ holds.

As shown in [2], the interval box $[x]$ with center in $[x_\infty]$ and radius

$$\sqrt{n_x \frac{\lambda_{min}}{\lambda_{max}}} \, d\left([x_\infty], [x_0]\right) \tag{15}$$

certainly belongs to the region of attraction of an *asymptotically stable* equilibrium x_∞. In (15), λ_{min} and λ_{max} are the minimum and maximum eigenvalues of P, resp. Furthermore, d is a function defined on $\mathbb{IR}^{n_x} \times \mathbb{IR}^{n_x}$ with

$$d : ([x], [y]) \mapsto \sup\{ r \in \mathbb{R} | \, B(r, [x]) \subset [y]\} \quad , \tag{16}$$

an interval box $[x] \subset \mathbb{IR}^{n_x}$, and $B(r, [x])$ denoting the set

$$\left\{ x \in \mathbb{R}^{n_x} \,\middle|\, \min_{a \in [x]} \|a - x\| < r \right\} \quad . \tag{17}$$

5 Robustness and Optimality

A general, interval-arithmetic framework for structure and parameter optimization of dynamical systems with both nominal and uncertain parameters has been presented in [8]. According to the definition of optimality for uncertain systems introduced therein, a control strategy is *optimal* if it leads to the *smallest upper bound* of the performance index for all possible $p \in [p]$. This is depicted exemplarily in Fig. 4 for l_{max} different control laws.

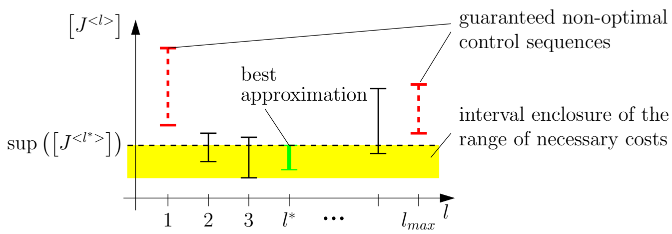

Figure 4: Optimality of control strategies with interval uncertainties.

These controls can either result from different parameterizations of a controller with a fixed structure (parameter optimization) or from different control structures (determined during structure optimization). In the latter case, piecewise constant and piecewise linear approximations of the globally optimal control sequence are determined.

In both cases, verification techniques are used to evaluate the integral performance index

$$J = f_{t_f}(x(t_f), p(t_f), t_f) + \int_0^{t_f} f_0(x(t), p(t), u(t), t)\, dt \tag{18}$$

to be minimized. The term $f_{t_f}(x(t_f), p(t_f), t_f)$ corresponds to terminal costs at the prescribed final point of time t_f if final states $x(t_f)$ are not specified exactly. The integrand $f_0(x(t), p(t), u(t), t)$ quantifies deviations between the current states and the desired trajectories of the state variables as well as the required control effort $u(t)$.

6 Optimal Control of a Mechanical Positioning System with Friction

In the remainder of this paper, optimal control of a mechanical positioning system with state-dependent switchings between different dynamical models is discussed [8]. For that purpose, a sliding mass m is considered. For a given initial position $x_1(0)$ and given initial velocity $x_2(0)$, an approximation of an optimal control strategy for the accelerating force $u(t) = F_{ext}(t)$ is determined under consideration of the friction force $F_f(x_2)$. The state equations for this system are given by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{m}(F_{ext}(t) - F_f(x_2)) \end{bmatrix} \quad \text{with} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} . \tag{19}$$

The friction characteristic is modeled by $l = 3$ discrete states $\mathcal{S} = \{S_1, S_2, S_3\}$, which are sliding friction for motion in "negative" (backward) direction ($= S_1$), static friction ($= S_2$), and sliding friction for motion in "positive" (forward) direction ($= S_3$).

In this friction model, interval parameters are considered for both the static friction coefficient $[F_s] = \left[\underline{F_s}\ ;\ \overline{F_s}\right]$ and the sliding friction coefficient $[\mu] = \left[\underline{\mu}\ ;\ \overline{\mu}\right]$. The resulting sliding friction force is then given by

$$F_f(x_2) = \begin{cases} -[F_s] + [\mu] \cdot x_2 & \text{for} \quad S_1 = \texttt{true} \\ +[F_s] + [\mu] \cdot x_2 & \text{for} \quad S_3 = \texttt{true} \end{cases} \tag{20}$$

and the static friction by

$$F_f(x_2) \in [F_s^{max}] = \left[-\overline{F_s}\ ;\ \overline{F_s}\right] \quad \text{for} \quad S_2 = \texttt{true}\ . \tag{21}$$

In Fig. 5, the results for an approximate solution of the optimal control problem are shown. If a piecewise constant approximation of the optimal control strategy is determined, the performance index

$$J = \int_0^{t_f} \left((x_1(t) - 1)^2 + x_2(t)^2 + u(t)^2 \right) dt + 100\Delta T \sum_{k=1}^{k_{max}} (u_k - u_{k-1})^2 \tag{22}$$

is minimized by a control strategy with a maximum number of 5 switchings in the considered time span. Analogously, the performance index

$$J = \int_0^{t_f} \left((x_1(t) - 1)^2 + x_2(t)^2 + u(t)^2 \right) dt + 100\Delta T \sum_{k=1}^{k_{max}} \dot{u}_k^2 \tag{23}$$

is minimized with $\dot{u}_k := \dot{u}(t)$, $t_{k-1} < t < t_k$ if piecewise linear control strategies are considered. Note that the absolute values of both performance indices cannot be compared directly. However, these criteria have been chosen to guarantee a maximum similarity between both types of approximations.

For the parameters $F_s = 0.015$, $\mu = 0.001$, $u \in [-1.00\ ;\ 1.00]$, and $\dot{u} \in [-0.50\ ;\ 0.50]$ it can be shown that the inequalities $J \leq 2.56$ and $J \leq 4.43$ hold for the piecewise constant and piecewise linear approximation, resp. In this example, the range of admissible final positions is described by the interval $x_1(t_f = 5) \in [0.9\ ;\ 1.1]$. The admissible final velocities are characterized by $x_2(t_f = 5) \in [-0.1\ ;\ 0.1]$.

7 Conclusions and Outlook on Future Research

In this paper, interval-based techniques for the verification of reachability, observability, and robust asymptotic stability of dynamical systems have been presented. They represent sufficient conditions for the corresponding system theoretic properties. Overestimation in the computation of guaranteed lower bounds of the rank of the reachability and observability matrices might lead to conservative results if large domains are considered for state variables and uncertain parameters. Efficient routines employing subdivision of those regions are available which help to express the resulting dependencies during rank computation as precisely as necessary. Together with the routine for verified stability analysis, they are included in a general-purpose framework for optimal and robust controller design. In this framework, also dynamical

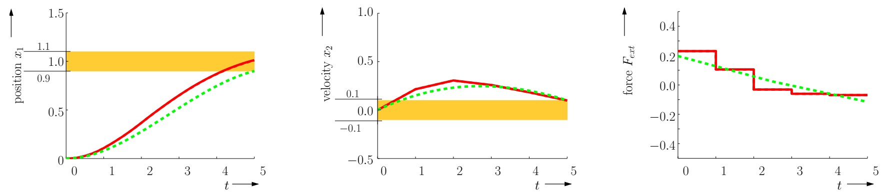

Figure 5: Approximate solution of the optimal control problem with state-dependent switchings. Solid lines: piecewise constant control, dashed lines: piecewise linear control.

systems with state-dependent switchings between different dynamical models can be considered.

In future work, we will develop extensions towards the design of state and disturbance estimators using interval techniques. These estimators will be used for the computation of worst-case bounds for the influence of approximation techniques in the design of both observers and controllers [7, 10]. A further step will be real-time state estimation in model-predictive control approaches for dynamical systems with interval uncertainties. For that purpose, the use of verified solvers for systems of differential-algebraic equations (DAEs) will be included and relations between criteria for solvability of DAEs and the criteria for controllability and observability of states in control theory will be investigated.

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