Interval-Related Talks at Econometric Conferences TES'2022 and ECONVN'2022

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Economics is a very important area of human activity. It is important to predict how the economy evolves, and to come up with measures that would help economy evolve in the desired direction. In these efforts, an important role is played by quantitative methods, known as *econometrics*. Many difficult-to-measure and difficult-to-predict factors affect the economy, which leads to a lot of uncertainty both in our knowledge of the current state of the economy and in our predictions of its future state. In particular, in many situations, we do not know the exact value of an economy-related quantity x, but we know a lower bound \underline{x} and an upper bound \overline{x} on this value, i.e., we know the interval $[\underline{x}, \overline{x}]$ that contains x. Because of this, econometricians have been analyzing issues related to interval uncertainty.

Several such papers were presented at two back-back econometric conferences held in January 2022: the 15th International Conference of Thailand Econometric Society TES'2023 (Chiang Mai, Thailand, January 5–7, 2022) and the 5th International Conference on Financial Econometrics ECONVN'2022 (Ho Chi Minh City, Vietnam, January 10–12, 2022); these papers will appear in post-conference edited books [1, 2]. These papers cover both elicitation and processing of interval-valued data.

The need for a special approach to elicitation of interval data comes from the fact that intervals – i.e., bounds – usually come from experts, and expert opinions are not perfect. Some experts are too cautious, they provide intervals which are too wide; other experts, vice versa, provide intervals which are too narrow and do not contain the actual value. It is therefore desirable to study, for each expert, the history of his/her interval estimates, and, based on this history, to learn how to correct these estimates. Several heuristic methods have been proposed for such a correction. Empirically, the most efficient method is when we keep the midpoint of the interval and multiply its radius (= half-width) by some constant c: smaller than 1 if we need to narrow down the original interval, and larger than 1 if we need to widen this interval. The paper "Correcting Interval-Valued Expert Estimates: Empirical Formulas Explained" by L. A. Berrout Ramos et al. provides a theoretical explanation for this empirical success: namely, it shows that this heuristic interval-to-interval function is the only one which is invariant with respect to such natural transformations as scaling (i.e., changing the measuring unit), shift (i.e., changing the starting point), and changing the sign (e.g., changing the viewpoint of analyzing debt from lender to borrower).

Several papers analyze how to take interval uncertainty into account when applying statistical data processing techniques. In some cases, we know a finiteparametric family of distributions, and we need to find the parameters of these distributions. For example, the constantly fluctuating stock prices can be viewed as normally distributed random variables with mean describing the true economic value of the corresponding stock and the variance describing speculationbased and/or uncertainty-motivated fluctuations. In particular, when we know an interval – e.g., an interval formed by the lowest and highest daily prices – both endpoints of this interval can be viewed as estimates of the true value. Techniques for processing such data are described in the paper "How to Find the Dependence Based on Measurements with Unknown Accuracy: Towards a Theoretical Justification for Midpoint and Convex-Combination Interval Techniques and Their Generalizations" by S. Chanaim et al.

Some families of distributions have a known theoretical justification – namely, the Central Limit theorem according to which, under some reasonable conditions, the distribution of a summary effect of many small independent factors is close to normal. If a family is both empirically confirmed and theoretically justified, we can confidently use it in data processing. In some cases, however, we have an empirical family that does not have a theoretical explanation. In this case, to increase our confidence, it is desirable to analyze the situation and understand why this family empirically appeared. In the paper "Distributions" on an Interval as a Scale-Invariant Combination of Scale-Invariant Functions: Theoretical Explanation of Empirical Marchenko-Pastur-Type Distributions" by V. Kreinovich, K. Alvarez, and Chon Van Le, such a theoretical explanation – also based on scale-invariance – is provided for an empirical family of distributions located on an interval.

Finally, in some cases, we do not a priori know a family of distributions. In this case, based on the available data, we need to estimate the distribution – e.g., by computing its cumulative distribution function (cdf) or its inverse function (whose values are known as quantiles). In situations when data comes with interval uncertainty, techniques for such estimation are presented in the paper "A Bayesian Approach to Quantile Regression for Interval-Valued Data" by R. Phadkantha, W. Yamaka, and S. Sriboonchitta.

References

 S. Sriboonchitta, V. Kreinovich, and W. Yamaka (eds.), Credible Asset Allocation, Optimal Transport Methods, and Related Topics, Springer, Cham, Switzerland, 2022, to appear. [2] Nguyen Ngoc Thach, V. Kreinovich, Doan Thanh Ha, and Nguyen Duc Trung (eds.), *Financial Econometrics: Bayesian Analysis, Quantum Uncertainty, and Related Topics*, Springer, Cham, Switzerland, 2022, to appear.