# Interval Papers at the 2022 Annual Conference of the North American Fuzzy Information Processing Society NAFIPS'2022

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**Fuzzy techniques and their relation to interval computations: in brief.** In many areas of human activity – be it medicine or driving – it is desirable to incorporate the knowledge and experience of best human specialists in an automated system that would help other make better decisions – or even make good decisions by itself. One of the challenges is in many cases, human experts describe their knowledge and experience by using imprecise ("fuzzy") natural-language words like "small". To incorporate this knowledge into computer programs, we need to translate this knowledge into computer-understandable numerical terms.

To perform such a translation, a natural idea is to ask the expert, for each possible value $x$ of the corresponding quantity, to mark, on some interval (e.g., interval $[0,1]$), the degree to which this value satisfies the given property (e.g., the degree to which this value is small). The resulting techniques – first proposed in the 1960s by Lotfi Zadeh – are known as *fuzzy techniques*, and the function $\mu(x)$ that assigns the degree to each value $x$ is known as a *membership function* or, alternatively, as a *fuzzy set*; see, e.g., [1, 4, 5, 7, 8, 11]. A fuzzy set can be alternatively described as a nested family of sets $\mathbf{x}(\alpha) \stackrel{\text{def}}{=} \{\boldsymbol{x} : \mu(x) \geq \alpha\}$ corresponding to different $\alpha > 0$. Such sets are known as the *$\alpha$-cuts* of the corresponding fuzzy set.

From the methodological viewpoint, fuzzy techniques – where everything is imprecise – seem to be opposite to interval techniques, where we are absolutely sure that the inputs to data processing are located in given intervals. However, from the computational viewpoint, processing fuzzy data naturally lead to interval problems.

For example, if we have membership functions $\mu_i(x)$ corresponding to quantities $x_1, \ldots, x_n$, then, for each data processing algorithm $y = f(x_1, \ldots, x_n)$, the natural membership function $\mu(y)$ has the form

$$\mu(y) = \max\{\min(\mu_1(x_1), \ldots, \mu_n(x_n)) : f(x_1, \ldots, x_n) = y\}.$$

This formula was first proposed by Zadeh and is thus called *Zadeh's extension* *principle.* It turns out that under reasonable conditions, each $\alpha$-cut of $y$ is the range of $y = f(x_1, \ldots, x_n)$ of $\alpha$-cuts of $x_i$:

$$\mathbf{y}(\alpha) = f\mathbf{x}_1(\alpha), \ldots, \mathbf{x}_n(\alpha)) = \{f(x_1, \ldots, x_n) : x_i \in \mathbf{x}_i(\alpha) \text{ for all } i\}.$$

For the typical case when all $\alpha$-cuts are intervals, we can thus use interval techniques to find this range.

Yet another relation between interval and fuzzy techniques is that the traditional fuzzy techniques implicitly assume that experts can describe their degree of certainty in different statements by an exact number. In reality, it is more reasonable to expect experts to provide only a rage (interval) of possible values. To process such interval-valued degrees, it is also natural to use interval techniques.

**Need for interval sessions at fuzzy conferences.** The relation between interval and fuzzy computations is well known. The problem is that fuzzy researchers are often unaware of the latest most efficient interval techniques and thus use outdated less efficient methods. To resolve this problem, many fuzzy conferences have special interval sessions for which one of the objectives is to explain the latest interval techniques.

These sections also help interval researchers to better understand the related computational problems.

**Papers presented at the conference.** In line with the above-described tradition, an interval session was organized at the 2022 Annual Conference of the North American Fuzzy Information Processing Society NAFIPS'2022 that was held in Halifax, Nova Scotia, Canada, May 31 – June 3, 2022.

Four papers were presented at this section. Papers [6] and [9] deal with the first of the two above-mentioned relations between interval and fuzzy – the need to process data under interval and fuzzy uncertainty. Specifically, [6] deals with solving differential equations under uncertainty, while the paper [9] describes how to best match courses and instructors in situations of uncertainty.

Papers [2] and [10] deal with the second relation – the need to consider interval-valued degrees. Specifically, [2] analyzes the relation between interval-values degrees and an alternative quantum-computing-motivated complex-valued approach. The paper [10] shows how to best adjust expert-provided interval-valued degrees so that the resulting system better matches the available data (e.g., of the actual expert driving).

Interval techniques were also used by several other papers presented at the conference.

**Thanks.** We want to use this opportunity to thanks all the authors and participants of the interval session and of the conference in general, and the conference organizers, especially Pawan Lingras, the conference General Chair, Scott Dick, Chair of the Program Committee, and – last but not the least – Raavee Kadam, Chair ot the Local Organizing Committee.

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