Interval-Related Papers at the 19th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems IPMU'2022

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Uncertainty is ubiquitous. We want to know the current state of the world, we want to know its future state, we want to know what to do to make this future state better for us. In all these objectives, we deal with numbers: to describe the current state or the future means to know the values of all numerical characteristics; to describe the design and/or a control also means to know the values of the corresponding parameters. To determine these numerical characteristics, we measure some quantities, we ask experts to estimate easy-to-estimate but difficult-to-measure quantities, and we use the known relations between quantities to estimate the values of other, difficult-to-measure and difficult-to-estimate quantities.

Neither measurements not expert estimates are absolutely accurate, both the measurement results and the expert estimates provide only an approximation to the actual (unknown) value. Because of this uncertainty, estimates based on the measurement results and expert estimates are also approximate. In short, uncertainty is ubiquitous.

It is important to take uncertainty into account. When we make decisions based on the available information, it is important to know the corresponding degree of uncertainty. For example, if our estimate of the asteroid trajectory shows that this asteriod will, in a few decades, get too close to Earth, then, depending on our level of uncertainty, we may be absolutely sure that it will fly by, or it may be that there is a possibility of a catastrophe – in which case we need to be prepared to divert it. In a more practical example, if we estimate the amount of oil in a given oilfield as 100 million tons, then if it is 100 ± 20, we need to start exploiting, but if it is 100 ± 200, maybe there is no oil at all – so we better perform more measurements before investing in the drilling.

Interval uncertainty is ubiquitous. In many practical situations, we know the bound Δ on the difference between our estimate $\widetilde{x}$ and the actual (unknown) value x of the corresponding quantity: $|\widetilde{x} - x| \le \Delta$. Based on this bound, we can conclude that the actual value x belongs to the interval $[\widetilde{x} - \Delta, \widetilde{x} + \Delta]$.

In some situations, this is all we know. In other cases, we have some information about the probabilities of different values x – e.g., we know that the difference $\widetilde{x} - x$ is normally distributed. In such cases, however, we rarely know the exact values of the parameters of the probability distributions – at best, we know these values with interval uncertainty.

In some situations, experts describe "fuzzy" degree of certainty in different possible values of the quantity x. Here too, it is difficult for an expert to come up with an exact value of this degree, it is much more natural for the expert to come up with an interval of possible values of the degree.

Interval-related talks at IPMU'2022. Because of this ubiquity, many talks at the 19th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems IPMU'2022 (Milan, Italy, July 11–15, 2022) dealt with interval uncertainty.

Several papers study the effect of interval uncertainty on *data processing*. The most widely used data processing techniques include *differential equations* – the usual tool for describing dynamics in physics, engineering, biology, and many other disciplines, and *probabilistic* techniques – the usual tool for describing uncertainty. Paper [1] and [11] analyze the effect of interval uncertainty on these two techniques.

An important part of data process is *aggregation* of different data points – e.g., results of different measurements and/or different expert estimates of the same quantity. Aggregation include different averaging techniques, "and"- and "or"-operations, etc. Interval-related aggregation techniques are studied in [3] and – for the case when many of the available data points come from malfunctioning sensors – in [6].

Aggregation deals with the case when we already know that the measurements and estimates correspond to similar situations. A related problem is using data to determine, based on the available uncertain data, how similar are the objects and/or situations – and whether they represent similar objects at all or similarity is simply an artifact of uncertainty (this is known as *aliasing problem* [5]). Interval-related similarity measures and their applications are studied in [9].

Paper [2] emphasizes the effect of interval uncertainty on general *decision making*. Specifically, when we make a decision, we want to select the optimal decision, i.e., the decision for which the value of the corresponding objective function is as large as possible (or, for some objective functions, as small as possible). Since we only know the values of all relevant quantities with interval uncertainty, the resulting values of the objective function are also only known with interval uncertainty. So, in some case, we have an alternative which is *necessarily* better than others – i.e., better for all possible values from the corresponding intervals – while in other cases, we have an alternative which is only *possibly* better: i.e., better for some values but not better for other values.

Specific applications of interval uncertainty include applications to detecting anomalies in crowdsourced work [12], to medical decision making [4], to psychology – where interval uncertainty explains why we usually overestimate small probabilities [7], to generating explainable summaries of online reviews [8], and to social decision making [10], where interval techniques are engaged to decide which use of the obsolete coastal military batteries in the city of Cartagena, Spain, will be the most beneficial for the city.

Last but not the least, an important paper [13] reminds us that while the same mathematics of interval techniques is used in different applications, it is important to understand and take into account differences – often subtle ones – between different applications of intervals. This paper illustrates this need on the example of intuitionistic fuzzy sets – where an expert provides the degree d_+ to which he/she supports the statement and the degree d_- to which he/she supports the opposite statement versus the interval-valued expert degree of support $[\underline{d}, \overline{d}]$. From the purely mathematical viewpoint, there is a natural 1-to-1 correspondence between these two types of uncertainty: $\langle d_+, d_- \rangle \Leftrightarrow [d_+, 1-d_-]$, but, as the authors show, these two uncertainty techniques lead, e.g., to different natual notions of similarity.

Overall, these results – and the remaining open problems – provide a good overview of current and potential uses of interval techniques in uncertainty processing and management.

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