Interval-Related Talks at the 2023 Annual Conference of North American Fuzzy Information Processing Society NAFIPS 2023 (Cincinnati, Ohio, May 31–June 2, 2023)

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At this conference, several papers dealt with interval uncertainty and interval computations. One of the papers – [3] – used interval computations to find the optimal investment in realistic situations when we only know future interest rates with interval uncertainty. Several other papers dealt with the fact that, in many practical problems, there are challenges related to applying traditional interval computations; namely:

- the problem of computing the range of a function $y = f(x_1, \ldots, x_n)$ under interval uncertainty – when we only know intervals $[\underline{x}_i, \overline{x}_i]$ of possible values of each quantity x_i – is, in general, NP-hard and thus, sometimes requires unrealistically long computation time;
- second, the resulting range interval is often too pessimistic: the actual interval of observed values of y is often much narrower.

The computational complexity of interval computations was emphasized in paper [2] that showed that for complex-valued data, the problem of computing the range is NP-hard already for single-use expressions, i.e., expressions line $x_1/(1 + x_2/x_3)$ in which each variable occurs only ones. This result contrasts to the real-valued case in which, for such expressions, the range can be computed by using straightforward interval computations, i.e., by replacing each elementary step in the computations by the corresponding operation of interval arithmetic.

One way to decrease the computation time is proposed in [1]: namely, the authors propose to only consider cases when each variable is equal to one of the endpoints. This idea leads to the exact range when the function f is monotonic with respect to each variable, and it leads to a reasonable estimate in several other practical examples.

The paper [5] deals with the need to come up with narrower intervals. Specifically, for the sum $y = x_1 + x_2$ of two quantities known with interval uncertainty $x_i \in [\underline{x}_i, \overline{x}_i]$, instead of the full range $[\underline{x}_1 + \underline{x}_2, \overline{x}_1 + \overline{x}_2]$ of the values of y, the authors consider an arithmetic average of this range and the narrowest possible range formed by the values $\underline{x}_1 + \overline{x}_2$ and $\overline{x}_1 + \underline{x}_2$ (corresponding to the case when x_1 and x_2 are perfectly anti-correlated). They show that the resulting operation is associative; they also show, on the example of a demographic problem, that this operation leads to reasonable results.

References

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[3] M. Mizukoshi, T. Mendonca da Costa, Y. Chalco-Cano, and W. A. Lodwick, “Interval sequence: choosing a sequence of the investment”, [4].

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[5] V. Wasques, A. Andrade, and P. Zanineli, “Associative property of interactive addition for intervals: application in the Malthusian model”, [4].