Interval-Related Talks at the World Congress of the International Fuzzy Systems Associatiom IFSA 2023 (Daegu, South Korea, August 20–23, 2023)

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Interval uncertainty is ubiquitous. In practice, we usually know the values of physical quantities with uncertainty: whatever estimate $\widetilde{x}$ we get from measurements or from experts is, in general, different from the (unknown) actual value x of this quantity.

Often, the only information that we have about the estimation error $\Delta x \stackrel{\text{def}}{=} \widetilde{x} - x$ is the upper bound Δ on its absolute value: $|\Delta x| \le \Delta$. In such situations, based on the estimate $\widetilde{x}$, the only information that we have about the actual value x is that x is contained in the interval $[\underline{x}, \overline{x}] = [\widetilde{x} - \Delta, \widetilde{x} + \Delta]$.

What do we do with (interval-valued) estimates? What do we usually do with measurement results (and expert estinates)?

First, we use them to estimate the values of related quantities y, in particular, to make predictions about future values of different physical quantities. These results we can present as numbers, or we can *visualize* them, to make it easier for us humans to understand.

In many practical situations, there is also a *second stage*, on which we use the results of the first stage to make decisions: namely, once we know the predicted outcomes of different actions, we select an alternative with the most beneficial outcome.

Sometimes, we also need a *preliminary stage*: namely, when we have several different estimates of the same quantity, we need to combine them into a single estimate.

What was presented at the conference. Several papers at this conference deal with problems appearing on all these stages.

Preliminary stage. Paper [2] deals with the *preliminary stage*: namely, it describes reasonable idea of how to handle the cases when different interval estimates are inconsistent.

First stage. Papers [3, 6, 7] deal with the problems related to the *first stage*, when we estimate the interval of possible value of different quantities y which are related to the directly estimated quantities $x_1, \ldots, x_n$ by a known relation $y = f(x_1, \ldots, x_n)$.

Paper [6] uses interval computations to predict reservoir inflows in extreme climate situations. Specific feature of such situations is that they are rare and thus, there is not enough data to accurately determine the corresponding probabilities, so interval uncertainty is an appropriate tool.

Paper [7] describes the need (and the possibility) to go beyond traditional interval computations. Namely, traditional interval computation techniques compute the full range on the corresponding function on the given intervals $[\underline{x}_i, \overline{x}_i]$, i.e., estimate the interval

$$[\underline{y}, \overline{y}] = \{f(x_1, \ldots, x_n) : x_1 \in [\underline{x}_1, \overline{x}_1], \ldots, x_n \in [\underline{x}_n, \overline{x}_n]\}.$$

This is the only way to get guaranteed estimates – which is important if we deal with critical situations where crossing some thresholds for y may be disastrous. However, in many practical applications, the resulting estimates are unnecessarily pessimistic: e.g., they consider the possibility that all input take extreme values at the same time, which is often highly improbable. To make more realistic estimates, the paper [7] first consider the most *optimistic* estimates – leading to the narrowest interval for y – and then finds *realistic* intervals as appropriate combinations of pessimistic and optimistic ones.

Paper [5] deals with *visualization* of the resulting interval uncertainty. For example, subintervals of the interval $[0, 1]$ can be represented as convex combinations of the intervals $[0, 0]$, $[1, 1]$, and $[0, 1]$. Thus, a natural way to visualize such an interval is to use colors – which are, in our perception, convex combinations of three basic colors: red, green, and blue. Intensity of the color can be used to describe how confident we are in the corresponding interval estimate.

Second stage. Finally, paper [1] deals with the second stage, i.e., with decision making based on interval data. Decision making under interval uncertainty is often a challenge, since while real numbers are naturally ordered, there is no natural total (linear) order between intervals. So, to make a decision based on interval-valued outcomes, we need to use some reasonable ordering of the intervals. If we want an automatic decision, we need a linear order; if we want a guidance to a human decision maker, then already a partial order will be helpful. For this purpose, the paper [1] describes all partial orders (on the set of all intervals) that satisfy some reasonable conditions.

References

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