A global optimization algorithm for Intlab

Tibor Csendes, László Pál, and Mihály Csaba Markót

University of Szeged, Sapientia University, and University of Vienna

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The Bound Constrained Global Optimization Problem

Consider the bound constrained global optimization problem

$$\min_{x \in X} f(x)$$

where the n-dimensional interval X is the search region, and $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ is the objective function. We assume that there exists at least one global minimizer point in X, that is also a stationary point. The considered algorithm is based on an inclusion function calculated by interval arithmetic or by other techniques.

Basic algorithm

$$
\begin{array}{l}
\text{GlobalOptimize } (f,\ X,\ \varepsilon,\ L_{res},\ f^*) \\
Y := X; \widetilde{f} := \overline{f}(m(X)); L_{res} := \{\}; L_{work} := \{\}; \\
\textbf{repeat} \\
\quad \text{OptimalComponent}(Y, k_1); \\
\quad \text{Bisection}(Y, k_1, U^1, U^2); \\
\quad \textbf{for } i := 1 \textbf{ to } 2 \textbf{ do} \\
\qquad f_U := f(U^i); \\
\qquad \textbf{if } \widetilde{f} < \underline{f_U} \textbf{ then next } i; \\
\qquad \textbf{if } \overline{f}(m(U^i)) < \widetilde{f} \textbf{ then } \widetilde{f} := \overline{f}(m(U^i)); \\
\qquad\qquad\qquad\qquad L_{work} := \text{CutOffTest}(L_{work}, \widetilde{f}); \\
\qquad \textbf{if } w(f_U) < \varepsilon \textbf{ then } L_{res} := L_{res} \cup (U, \underline{f_U}); \\
\qquad\qquad\quad \textbf{else } L_{work} := L_{work} \cup (U, \underline{f_U}); \\
\quad \textbf{if } L_{work} \neq \{\} \textbf{ then } Y := \text{Head}(L_{work}); \\
\textbf{until } L = \{\}; \\
Y := \text{Head}(L_{res}); f^* := [\underline{f_Y}, \widetilde{f}]; \\
\textbf{return } L_{res}, f^*;
\end{array}
$$

Additional accelerating devices

- monotonicity test
- concavity test
- interval Newton step
- a centered form inclusion function

We are about to implement the pf^*-based new heuristic rules, and a fitting combination of them.

Use

```
>> startintlab
```

```
>> [intv, min, stats] = GOP(FUN, b, tol);
```

The content of the file `sh5.bnd` is:

```
S5
4
0 10
0 10
0 10
0 10
1e-8
```

A typical result

```
Function name:  S5
The set of global minimizers is located in the union of
the following boxes:

[4.00003713662883, 4.00003718945147],
[4.00013323800906, 4.00013329348396],
[4.00003713910016, 4.00003717168197],
[4.00013326916774, 4.00013328566954]

The global minimum is enclosed in:
[-10.153199679058694, -10.15319967905819 9]

Statistics:

Iter Feval Geval Heval MLL CPUt(sec)
16    126    86     7   10     6.69
```

SIAM 100 \$, 100 Digits Challenge

Another telling example is the #4 problem from the set of the SIAM 100\$, 100 Digits Challenge announced in 2002 (ten exact digits were to be determined for each of the ten numerical problems). The function to be minimized was:

$$\exp(\sin(50x))+\sin(60e^y)+\sin(70\sin(x))+\sin(\sin(80y))-\sin(10(x+y))+\frac{1}{4}(x^2+y^2).$$

We run our INTLAB based interval global optimization algorithm for the search interval $[-10,10]^2$. The parameter file was:

```
SIAM
2
-10 10
-10 10
1e-8
```

SIAM 100 $, 100 Digits Challenge result

```
function y = siam(x)
y = exp(sin(50*x(1)))+sin(60*exp(x(2)))+sin(70*sin(x(1)))+
sin(sin(80*x(2)))-sin(10*(x(1)+x(2))) + 1/4 * (x(1)^2+x(2)^2);
```

The obtained result was:

```
The set of global minimizers is located in the union of
the following boxes:
c1:  [-0.02440308068263, -0.02440307781118]
     [ 0.21061242712377,  0.21061242717589]
The global minimum is enclosed in:
[-3.306868647475258 4, -3.306868647475228 7]
Statistics:
Iter Feval Geval Heval MLL CPUt(sec)
238  1723  1151    90  75  75.63
```

Computational test results / basic algorithm

		C-XSC code		INTLAB code	
Problem	Dim	NIT	NFE	NIT	NFE
S5	4	84	307	83	305
S7	4	259	864	259	864
S10	4	310	1,016	313	1025
THCB	2	5,591	16,779	5,591	16,779
BR	2	149	480	149	480
RB2	2	75	250	74	247
RB5	5	2,339	7,063	2,339	7,063
L8	3	21	81	21	81
L9	4	28	109	28	109
L10	5	35	137	35	137
L11	8	141	477	141	477
L12	10	412	1,455	412	1,455
L13	2	22	81	22	81
L14	3	35	131	35	131
L15	4	52	194	52	194
L16	5	72	270	72	270
L18	7	614	2,100	614	2,100
Schw2.1	2	308	933	308	933
Schw3.1	3	32	119	31	117
Schw2.5	2	72	232	72	232
Schw2.14	4	924	3,011	924	3,011
Schw2.18	2	5,623	17,093	5,623	17,093
Schw3.2	3	110	355	106	342
Schw3.7_5	5	191	605	191	605
Griew7	7	216	729	216	729
R4	2	1,547	5,137	1,547	5,137
R5	3	355	1,235	355	1,235
R6	5	1,939	6,695	1,939	6,695

Computational test results, basic algorithm /2

		C-XSC code		INTLAB code	
Problem	dim	MLL	CPU	MLL	CPU
S5	4	14	0.02	13	14.20
S7	4	43	0.06	43	55.78
S10	4	58	0.17	55	93.73
THCB	2	1,128	0.36	1,128	178.58
BR	2	18	0.00	18	6.64
RB2	2	10	0.00	10	1.72
RB5	5	56	0.33	56	167.84
L8	3	8	0.00	8	1.94
L9	4	11	0.00	11	3.41
L10	5	14	0.00	14	5.22
L11	8	23	0.10	23	28.23
L12	10	44	0.71	44	111.63
L13	2	7	0.00	7	1.45
L14	3	10	0.00	10	3.09
L15	4	13	0.00	13	5.69
L16	5	16	0.01	16	9.55
L18	7	67	0.35	67	98.92
Schw2.1	2	44	0.00	44	11.63
Schw3.1	3	7	0.00	6	1.89
Schw2.5	2	7	0.00	7	1.44
Schw2.14	4	82	0.06	82	32.19
Schw2.18	2	678	0.48	678	104.73
Schw3.2	3	13	0.00	12	3.80
Schw3.7_5	5	32	0.01	32	6.59
Griew7	7	43	0.07	43	27.44
R4	2	348	0.11	348	51.39
R5	3	70	0.03	70	29.44
R6	5	226	0.48	226	264.31

Computational test results, sophisticated algorithm

		Old, C-XSC code			New, INTLAB code		
Problem	Dim	NIT	NFE	NGE	NIT	NFE	NGE
S5	4	16	126	86	16	126	86
S7	4	18	129	84	17	121	78
S10	4	18	126	81	17	123	78
H3	3	42	184	135	42	184	135
H6	6	217	1,014	735	220	1,038	756
GP	2	2,351	15,314	9,430	2,351	15,319	9,427
SHCB	2	130	694	455	130	694	455
THCB	2	56	327	229	56	327	229
BR	2	52	282	200	52	282	200
RB	2	43	263	172	43	263	172
RB5	5	612	4,907	3,685	607	4,878	3,664
L3	2	293	1,890	1,301	293	1,890	1,301
L5	2	88	578	397	88	578	397
Schw2.1	2	226	1,312	951	226	1,312	951
Schw3.1	3	14	91	61	14	91	61
Schw2.5	2	53	307	216	53	307	216
Schw2.14	4	369	2,600	1,820	408	2,780	1,913
Schw2.18	2	51	284	201	51	284	201
Schw3.2	3	33	201	135	25	164	110
Schw3.7_5	5	129	677	484	129	677	484
Schw3.7_10	10	7,566	35,385	25,771	7,566	35,385	25,771
Griew5	5	691	9,854	6,424	705	9,940	6,482
Griew7	7	40	272	163	40	272	163
R4	2	154	902	648	153	899	645
R5	3	173	1,555	1,174	173	1,555	1,174
R6	5	227	2,255	1,826	227	2,255	1,826
R7	7	380	4,437	3,711	380	4,437	3,711
R8	9	471	6,170	5,266	471	6,170	5,266
EX2	5	41,794	250,885	177,929	14,774	89,318	65,862

Computational test results, sophisticated algorithm /2

		Old, C-XSC code			New, INTLAB code		
Problem	dim	NHE	MLL	CPU	NHE	MLL	CPU
S5	4	7	10	0.01	7	10	10.14
S7	4	7	14	0.03	6	14	13.05
S10	4	6	16	0.03	6	17	18.56
H3	3	3	12	0.01	3	12	11.20
H6	6	25	69	0.33	27	69	113.47
GP	2	608	480	0.68	608	480	630.33
SHCB	2	25	51	0.01	25	51	21.06
THCB	2	22	19	0.00	22	19	8.11
BR	2	18	12	0.00	18	12	6.91
RB	2	15	11	0.00	15	11	3.17
RB5	5	411	77	0.45	410	73	220.30
L3	2	97	138	0.16	97	138	115.11
L5	2	28	31	0.04	28	31	41.06
Schw2.1	2	88	26	0.02	88	26	36.78
Schw3.1	3	5	6	0.00	5	6	2.67
Schw2.5	2	28	5	0.00	28	5	4.02
Schw2.14	4	179	78	0.09	190	65	76.53
Schw2.18	2	22	8	0.00	22	8	3.81
Schw3.2	3	12	9	0.00	11	7	3.34
Schw3.7_5	5	32	32	0.03	32	32	25.98
Schw3.7_10	10	1,024	1,024	11.35	1,024	1,024	2,585.11
Griew5	5	597	32	1.04	611	32	575.95
Griew7	7	7	52	0.05	7	52	20.63
R4	2	51	39	0.02	51	39	17.72
R5	3	109	43	0.10	109	43	75.22
R6	5	143	29	0.39	143	29	186.75
R7	7	257	47	1.59	257	47	526.50
R8	9	321	65	3.81	321	65	951.27
EX2	5	19,124	1,969	72.93	6,928	1,610	12,042.27

References

T. Csendes: New subinterval selection criteria for interval global optimization. J. Global Optimization 19(2001) 307-327.

T. Csendes and L. Pál: A basic interval global optimization procedure for Matlab/INTLAB. Proc. of the NOLTA2008 conf., Budapest, IEEE, 2008.

T. Csendes, L. Pál, J.O.H. Sendín, J.R. Banga: The GLOBAL Optimization Method Revisited. Optimization Letters 2(2008) 445-454.

L. Pál and T. Csendes: A global optimization algorithm for Intlab. Optimization Methods and Software, submitted.

S.M. Rump, Intlab – Interval Laboratory. In: T. Csendes (ed.): Developments in Reliable Computing, Kluwer Academic Publishers, Dordrecht, 1999, 77-104.

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The new Matlab/INTLAB based interval global optimization algorithm will also be available soon as a part of the GLOBAL package. The latter is to be downloaded from

www.inf.u-szeged.hu/∼csendes.