KATHOLIEKE UNIVERSITEIT LEUVEN

Mechanical Engineering - Departement Werktuigkunde

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Stress analysis of a doubly reinforced concrete beam with uncertain structural parameters

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1. Introduction
   - Non-deterministic modelling in engineering
   - Fuzzy uncertainty analysis in engineering
   - Implementation strategies
2. Response surface approaches
   - Objective
   - Taylor expansion response approximation
   - Kriging surrogate modelling approach
3. Application: doubly reinforced concrete beam
   - Description of the deterministic model
   - Introduction of uncertainty in the model
   - Results of the response surface approaches
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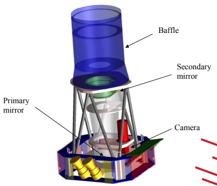
Non-determinism in modelling

Every aspect in the numerical model or solution procedure that introduces doubt or scatter on the outcome of the analysis

- in design procedures
  - present in all phases of design process
  - appearing in different forms
- has to be taken into account
  $\longrightarrow$ represented using non-deterministic mathematical concepts
- parametric approach: uncertainty on parameters is processed to uncertainty on output quantities

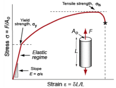
Introduction

Non-deterministic modelling in engineering

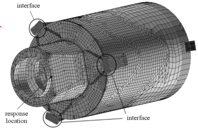


abstraction of environment
loads, connections, ...

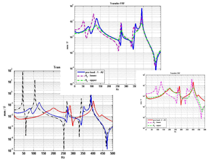
quantification of model properties
geometry, material, ...



SCATTER



numerical procedure
discretisation, numerical properties, ...



- scatter: caused by variation in design (*variability, aleatory*)

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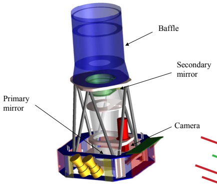
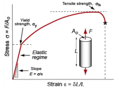
Non-deterministic modelling in engineering

COROT

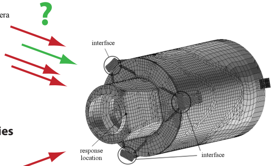


abstraction of environment
loads, connections, ...

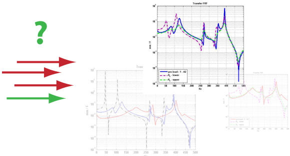
quantification of model properties
geometry, material, ...



DOUBT



numerical procedure
discretisation, numerical properties, ...



- scatter: caused by variation in design (*variability, aleatory*)
- doubt: caused by uncertainty of designer (*uncertainty, epistemic*)

Introduction

Non-deterministic modelling in engineering

different tools for dealing with parametric non-determinism:

- variability
  - sampling methods
  - random processes
  - stochastic methods
  - ...
- uncertainty
  - interval methods
  - fuzzy approach
  - interval probabilities
  - info-gap
  - ...

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Non-deterministic modelling in engineering

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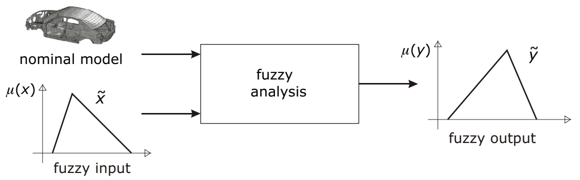
Fuzzy uncertainty analysis

- input: membership functions of fuzzy model properties
- output: membership function of output quantity

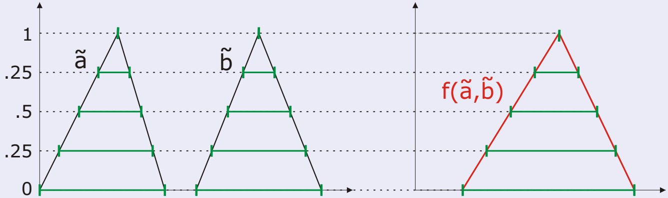


- implementation: using α-sublevel technique
- added value in design procedures
  - definition of tolerances
  - design for robustness

Introduction

Fuzzy uncertainty analysis in engineering

α-level strategy



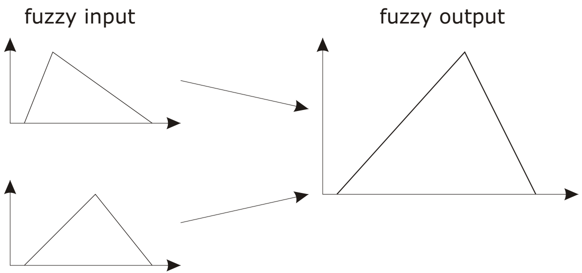
- interval analysis is core of fuzzy analysis
- selection of number of α-levels is a trade-off between accuracy and numerical cost

Introduction

Fuzzy uncertainty analysis in engineering

Definition of tolerances

- interval concept very appropriate representation of tolerances in early design stage
- derive allowable tolerance intervals:



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Fuzzy uncertainty analysis in engineering

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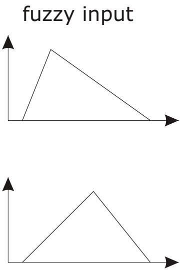

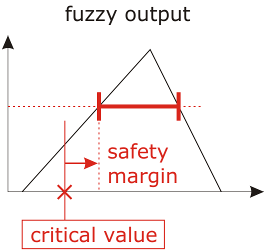

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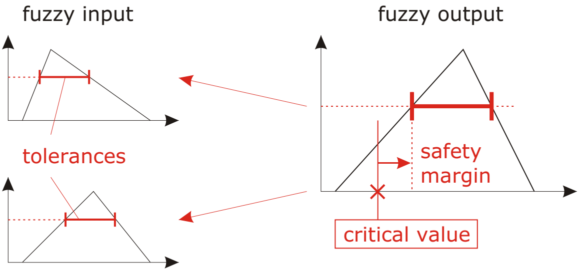

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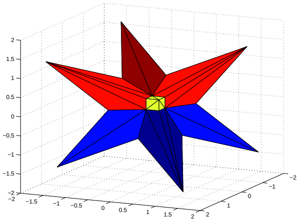
Implementation strategies

Basic interval problem

If deterministic FE analysis is represented by $f(\boldsymbol{x})$, uncertain parameters bounded by $\boldsymbol{x}^{\boldsymbol{I}}$, find the set of possible outputs y:

$$\boldsymbol{y}^{\boldsymbol{S}} = \left\{ \boldsymbol{y} \mid \left(\boldsymbol{x} \in \boldsymbol{x}^{\boldsymbol{I}} \right) \left(\boldsymbol{y} = f(\boldsymbol{x}) \right) \right\}$$

output set $\boldsymbol{y}^{\boldsymbol{S}}$ can adopt any form in output space



$\rightarrow$ hypercubic approximation

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Implementation strategies

interval arithmetic

- approaching the exact interval result from outside
- based on interval arithmetics
- can be subject to conservatism due to dependency problem

optimization

- approaching the exact interval result from inside

$$\underline{y}_i = \min_{\boldsymbol{x} \in \boldsymbol{x}^I} f_i(\boldsymbol{x}), \; i = 1 \ldots n$$

$$\overline{y}_i = \max_{\boldsymbol{x} \in \boldsymbol{x}^I} f_i(\boldsymbol{x}), \; i = 1 \ldots n$$

- global optimization $\longrightarrow$ computationally expensive
- approximate optimization schemes
  - DOE: full factorial (vertex method), 3-level FF, ...
  - local series expansion about interval mid-value
  - using global surrogate models

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Response surface approaches

Objective

principal idea RSA for interval analysis

1. to build a surrogate model based on limited information of the goal function $f(\boldsymbol{x})$
2. to use knowledge from this surrogate model to speed up optimization

Two methods are used in this work:

1. Taylor expansion in interval mid-point to perform monotonicity check (sensitivity analysis)
   see also A. Pownuk, "General Interval FEM Program Based on Sensitivity Analysis", NSF workshop on Reliable Engineering Computing, February 20-22, 2008, Savannah, Georgia, USA, pp.397-428
2. Kriging approach for global response surface modelling
   see also M. De Munck, D. Moens, W. Desmet, and D. Vandepitte, "A kriging based optimization algorithm for interval and fuzzy frf analysis," in 8th. World Congress on Computational Mechanics (WCCM8), (Venice), 2008

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Response surface approaches

Taylor expansion: procedure

1. build first order approximation of partial derivative of output function in mid-point of the uncertainty space

$$\frac{\partial f(x)}{\partial x_i} \approx \frac{\partial f(x_0)}{\partial x_i} + \sum_j \frac{\partial f^2(x_0)}{\partial x_i \partial x_j}(x_j - x_{j0})$$

2. evaluate partial derivatives in vertex points

$$\left(\frac{\partial f}{\partial x_i}\right)^l \approx \frac{\partial f(x_0)}{\partial x_i} - \sum_j \left|\frac{\partial f^2(x_0)}{\partial x_i \partial x_j}\right| \Delta x_j$$

$$\left(\frac{\partial f}{\partial x_i}\right)^u \approx \frac{\partial f(x_0)}{\partial x_i} + \sum_j \left|\frac{\partial f^2(x_0)}{\partial x_i \partial x_j}\right| \Delta x_j$$

where $\Delta x_j = \frac{x_j^u - x_j^l}{2}$

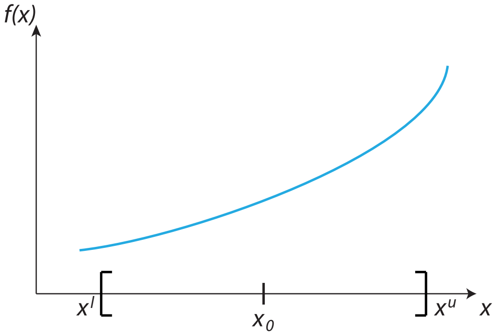
3. condition for monotonicity

$$0 \notin \left[\left(\frac{\partial f}{\partial x_i}\right)^l, \left(\frac{\partial f}{\partial x_i}\right)^u\right]$$

4. if monotonicity is detected, function is evaluated at optimal vertex combinations

Response surface approaches

Taylor expansion



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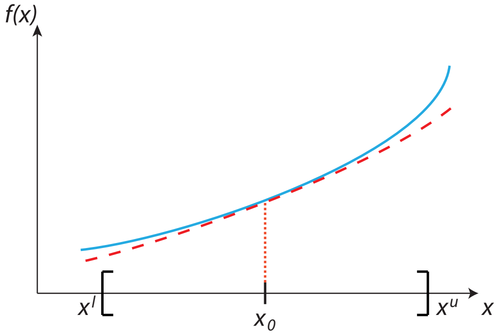
Taylor expansion



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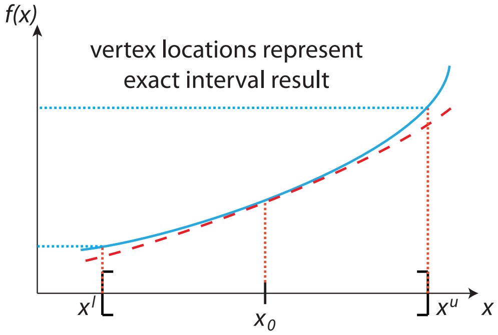

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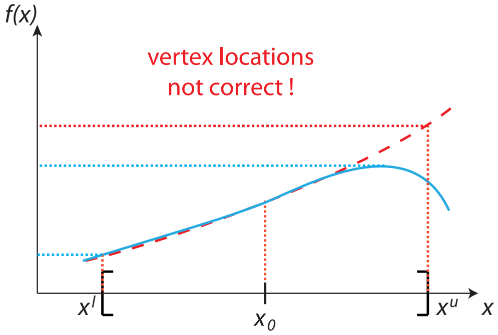
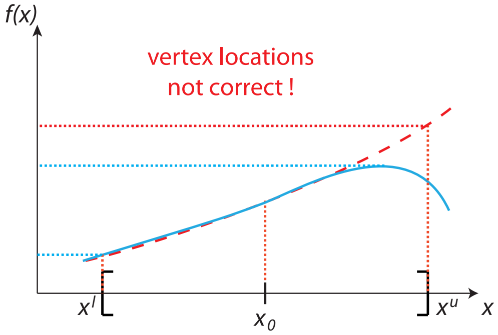


Response surface approaches

Taylor expansion



- properties
  - + very efficient calculation
  - + improved confidence in vertex results
  - + reliable for low order behavior
  - - does not capture higher order behavior
  - - no solution when non-monotonicity is detected

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Response surface approaches

Kriging approach: principle

- Kriging (DACE) approximation
- objective: build a reliable response surface with a limited number of samples
- principle idea: errors are no longer considered independent
  - model:

$$\tilde{f}(x_i) = \sum_{k=1}^{n} a_k \tilde{f}_k(x_i) + \epsilon(x_i)$$

  - correlation:

$$\mathrm{Corr}\big(\epsilon(x_i), \epsilon(x_j)\big) = \exp\big(-d(x_i, x_j)\big)$$

$$d(x_i, x_j) = \sum_{h=1}^{k} \theta_h \left|x_{i_h} - x_{j_h}\right|^{p_h}$$

  - DACE stochastic process model

$$\tilde{f}(x_i) = \mu + \epsilon(x_i)$$

Response surface approaches

Kriging approach

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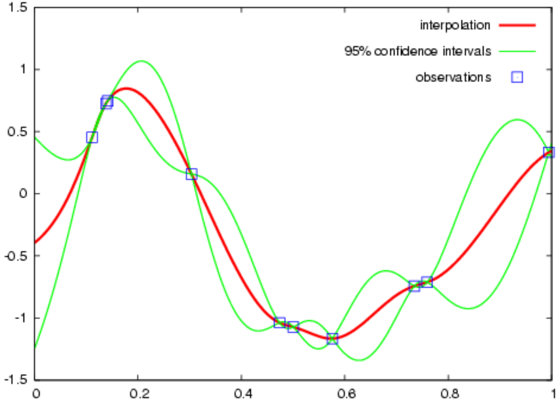
Kriging model: example



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Kriging approach: procedure

1. build initial Kriging RS $\tilde{f}(\boldsymbol{x})$ based on space filling design
2. repeat until convergence:
   - generate set of candidate points $\boldsymbol{x}_i$ for improved RS, and calculate $\tilde{f}(\boldsymbol{x}_i)$ and maximum error $\Delta\tilde{f}(\boldsymbol{x}_i)$
   - search for point(s) with the maximum expected improvement

$$\min_i \left[\tilde{f}(\boldsymbol{x}_i) - \Delta\tilde{f}(\boldsymbol{x}_i)\right]$$

$$\max_i \left[\tilde{f}(\boldsymbol{x}_i) + \Delta\tilde{f}(\boldsymbol{x}_i)\right]$$

   - rebuild $\tilde{f}(\boldsymbol{x})$ by adding selected point(s) to Kriging RS
3. perform optimization on $\tilde{f}(\boldsymbol{x})$

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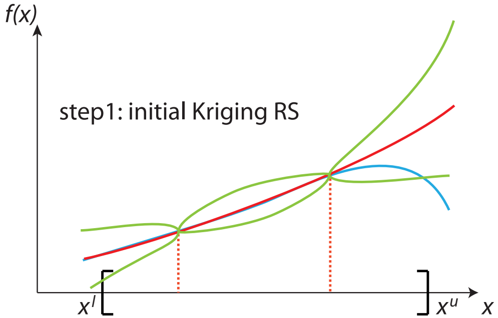

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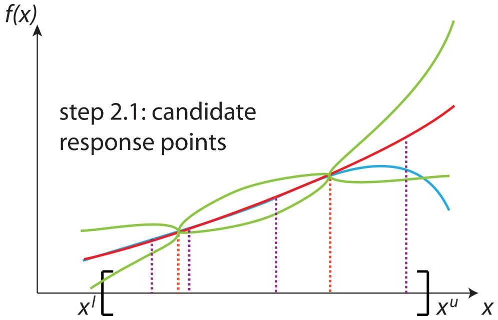

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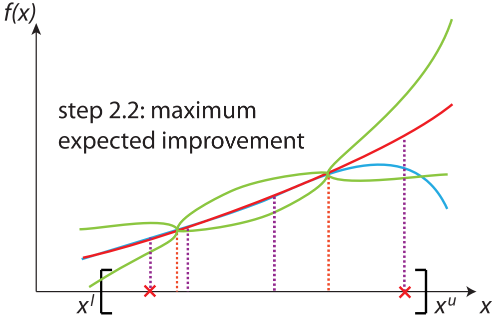

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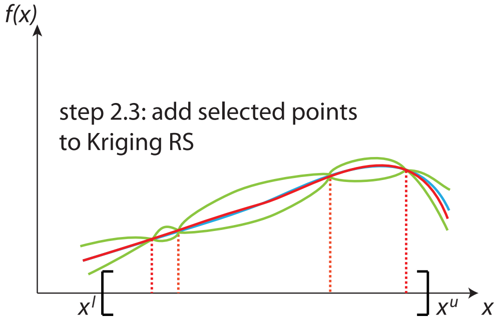

- properties
  - \+ requires limited number of sampling points
  - \+ finds local extrema
  - \+ easily extendable towards multi-objective optimization
  - \+ surface can be re-used at different α-levels
  - \- extrema evaluated on surrogate model

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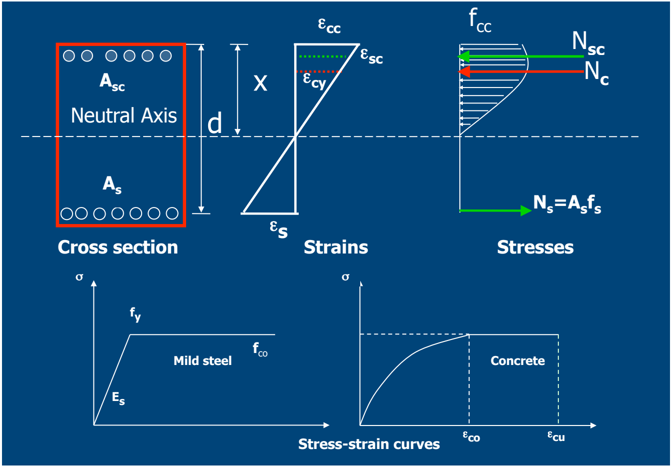
Doubly reinforced beam

Doubly reinforced beam: deterministic case

- objective: given geometry and material properties, determine the allowable external bending moment



Doubly reinforced beam

Deterministic model

Doubly reinforced beam: deterministic case

→ implicit non-linear problem solved using deterministic iterative solution scheme on ϵ_{cc}

- for increasing ϵ_{cc}
  1. express all forces N_{sc}, N_s and N_c as a function of x
  2. horizontal equilibrium yields quadratic equation in x:

$$Ax^2 + Bx + C = 0 \quad (1)$$

     with $A(\epsilon_{cc})$, $B(A_S, A_{SC}, E_S)$ and $C(A_S, A_{SC}, E_S)$
  3. from x, the location of the resultant compression force in concrete is determined
  4. finally, the internal moment of resistance M_R is determined
- stop when either ϵ_{cc}, f_s or f_{sc} reaches physical limit

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Doubly reinforced beam

Uncertain model

Doubly reinforced beam: problem with uncertainty

- fuzzy uncertainty has been defined on geometric and material properties of steel: A_S,A_{SC},E_S
- triangular fuzzy membership functions, with a base interval $[-5\%, +5\%]$
- objective: given an external bending moment, determine the tolerances on geometry and the allowable interval on the Young's modulus

→ solved using interval arithmetic approach, Taylor and Kriging approach

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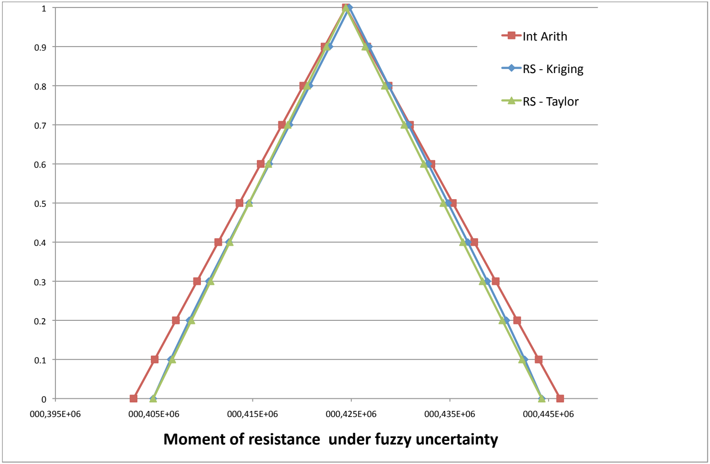
Results



→ Taylor expansion: $1 + 2 \times 10 = 21$ function evaluations
→ Kriging approach: 20 function evaluations

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Conclusions

- 2 response surface methodologies were presented to approximate interval solutions in numerical analysis
- the Taylor approach proved to be a very efficient calculation scheme, resulting in improved confidence in vertex results
- the Kriging approach is applicable for non-monotonic functions, requiring a minimal amount of sampling points
- for the doubly reinforced concrete beam problem, both methods yield very accurate interval results