A Differential Inequalities Method for Verified Solution of IVPs for ODEs using Linear Programming for the Search of Tight Bounds

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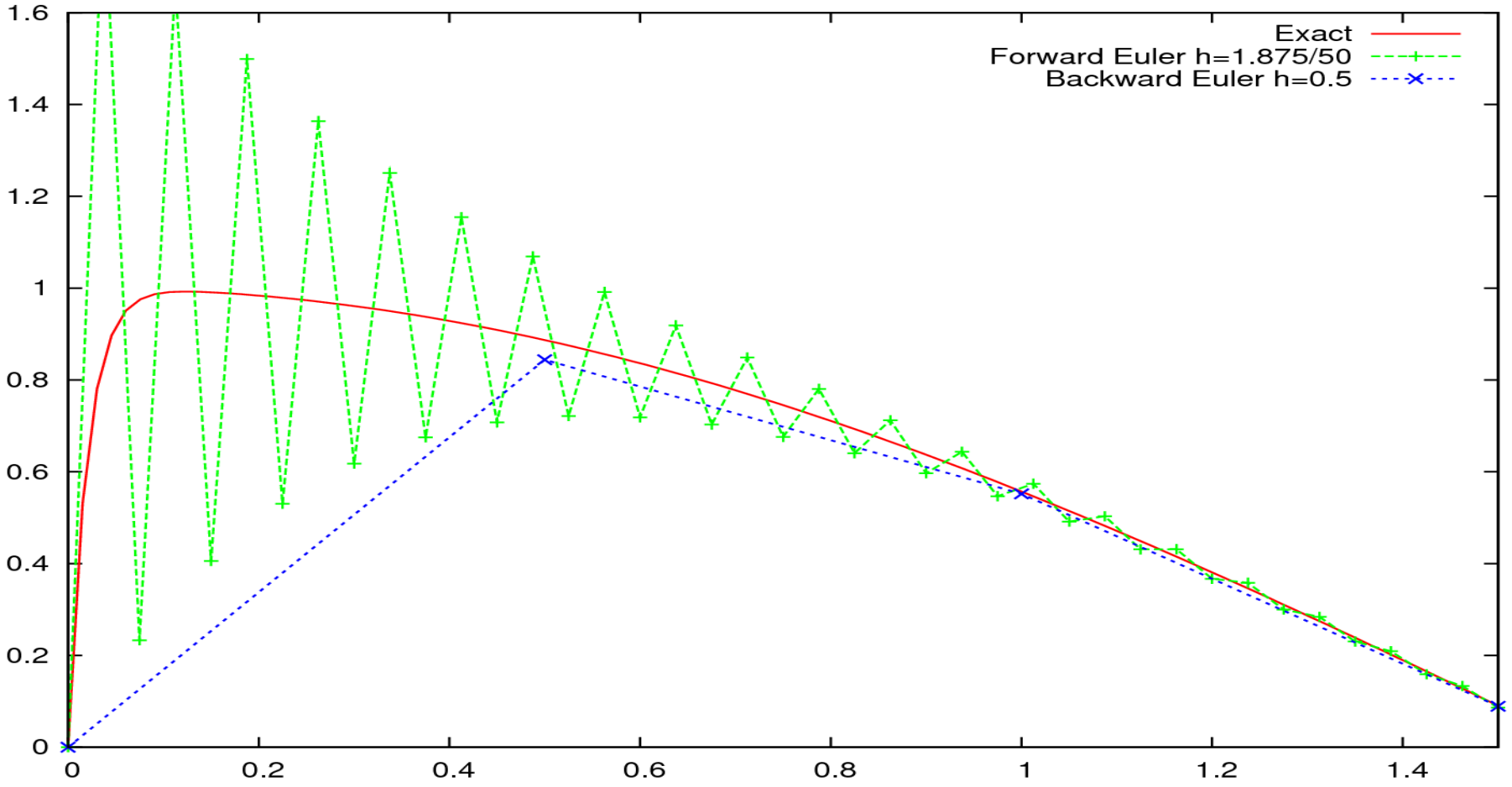
Stiff IVP Example by Curtiss & Hirschfelder

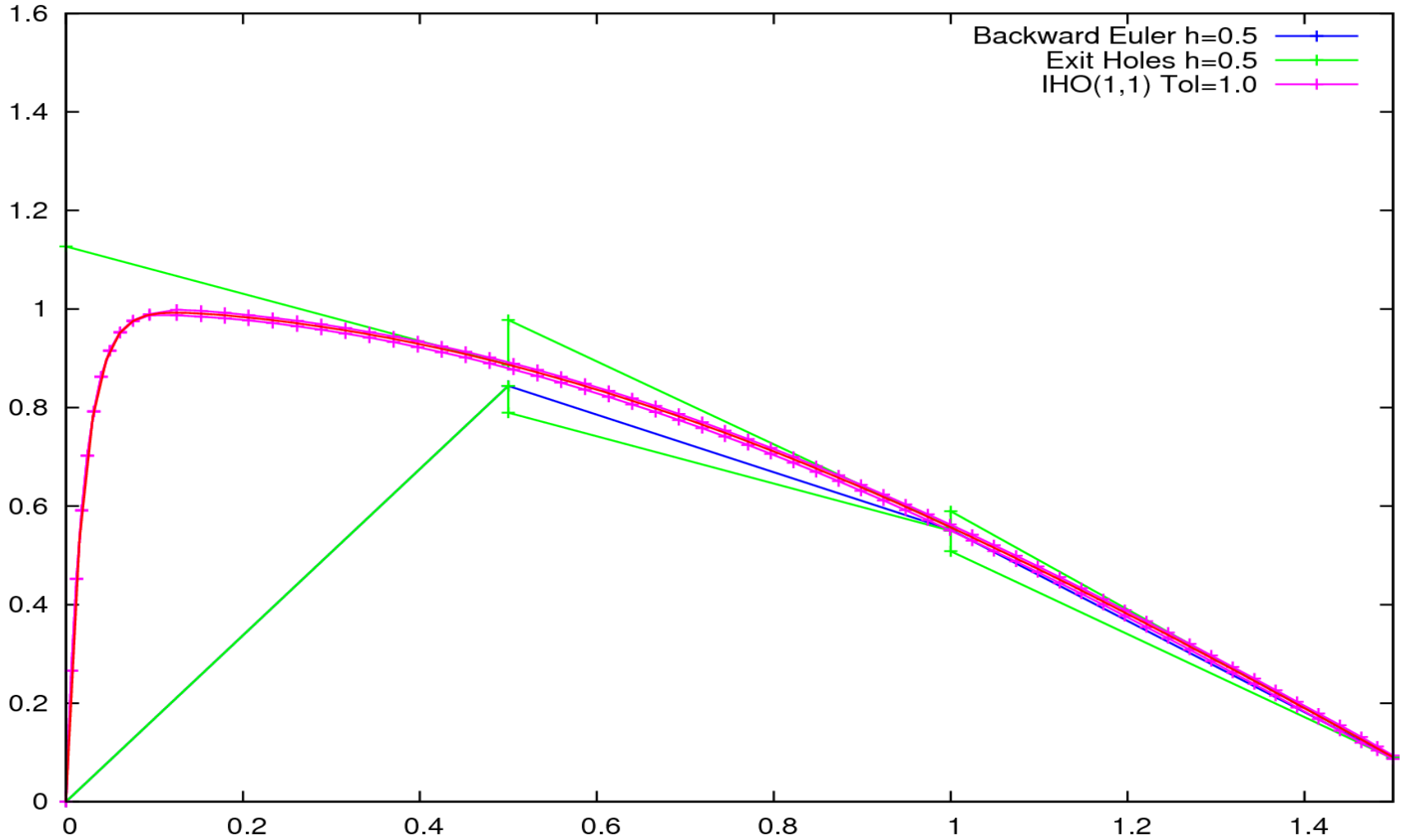
$$\dot{x} = 50(\cos(t) - x)$$

$$x(0) = 0$$

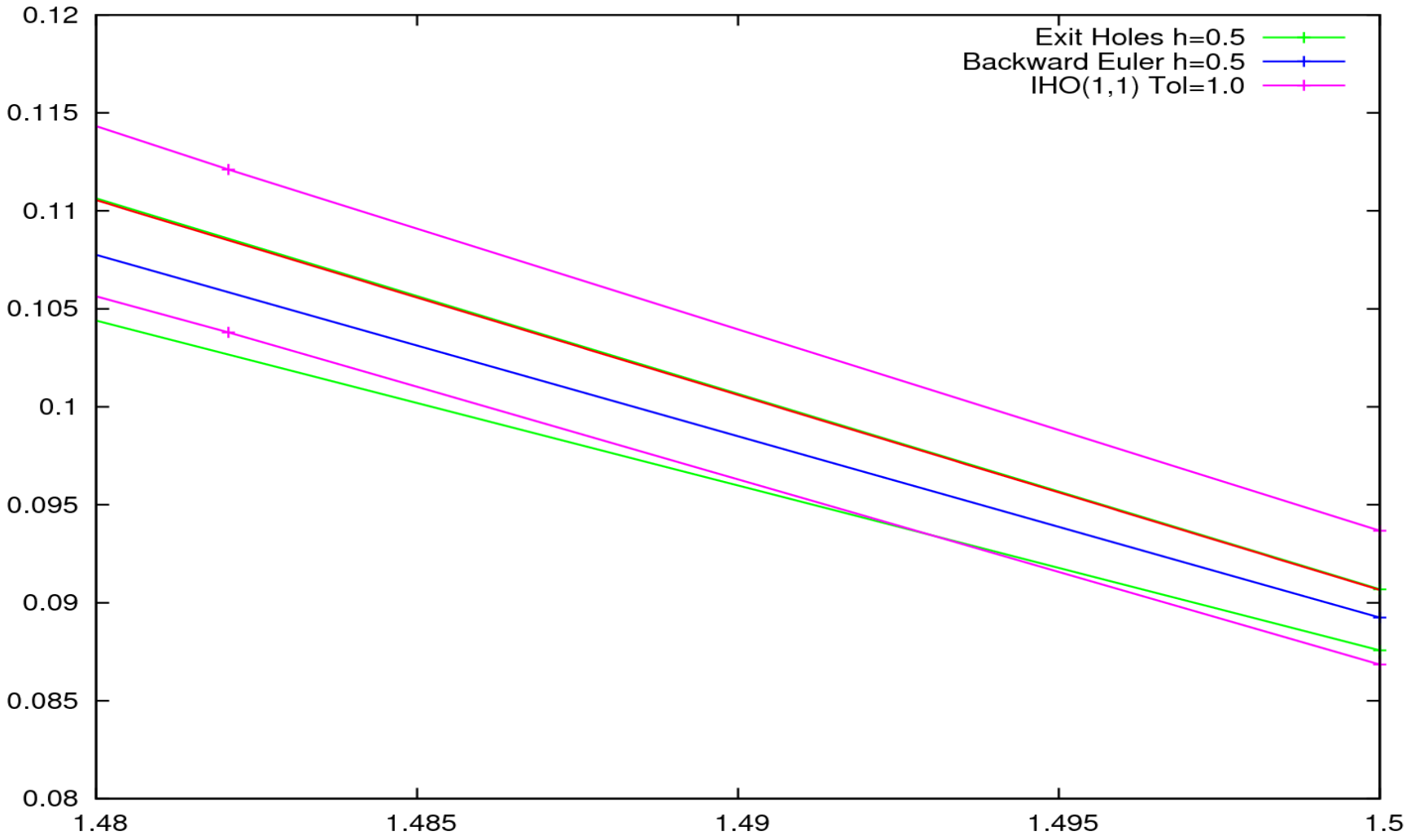
Forward and Backward Euler Point Solutions of Curtiss & Hirschfelder IVP



IHO and Exit Holes Interval Solutions of Curtiss & Hirschfelder IVP



Interval Solutions near the final time

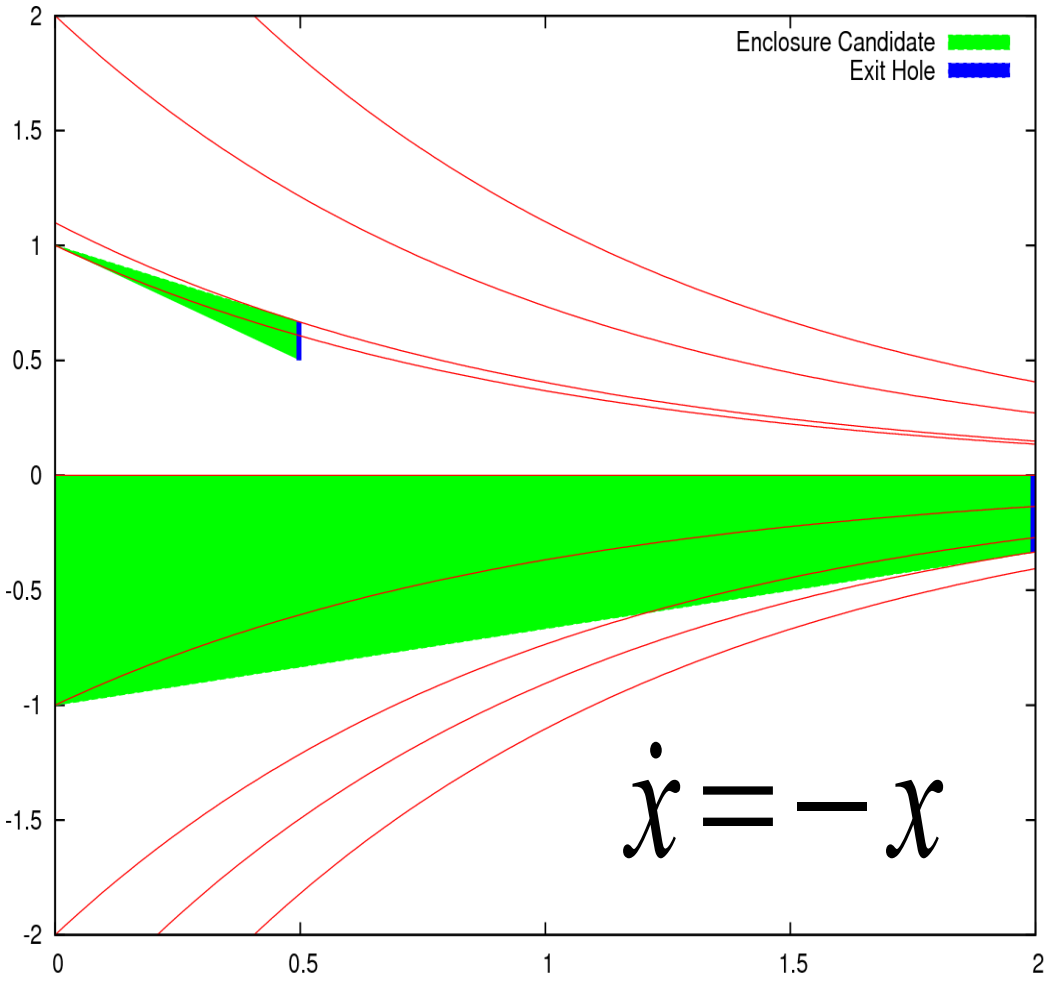


Works on Interval Solution of IVP

- N. Nedyalkov – Interval Hermite-Obreshkov method
- Kalmykov,Shokin – Chaplygin inequalities
- A. Neumaier – logarithmic norm for realistic bounds of dissipative ODE
- Gennat, Tibken – Muller theorem
- Berz, Makino – Taylor Models

Exit Hole from Intersection

Use of Müller Theorem for Scalar Example



Searching Trapezoid by LP

Search for better Trapezoid using Linear Programming

$$\underline{k}_i \leqslant F_i(y_c) + HULL([J^{il}](S^{ilL} - y_c), [J^{il}](S^{ilR} - y_c))$$

- Have an exit hole from the previous step
- Choose time interval
- Guess sufficient large a priori box Yil and Yih for each coordinate (they may be the same), compute J
- Choose the center from point
- Now defect constraint is a convex piecewise linear function
- Optimization target is minimization of hole width

Searching Trapezoid by LP

Size of LP problem

$$\underline{k}_i \leqslant F_i(y_c) + HULL([J^{il}](S^{ilL} - y_c), [J^{il}](S^{ilR} - y_c))$$

- 4*n variables for x and k
- 4*nz variables for interval products for each nonzero Jacobian
- 8*nz constraints modelling interval products
- 4*n constraints modelling HULL
- 2*n or 4*n constraints to cover previous exit hole
- Total 4*(n+nz) variables and 8*(n+nz) constraints

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