Derivative-Free Constraint-Driven Global Optimization

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The Optimization Paradigm

Choose *decision variables* so as to
maximize (or minimize) the *objective function*
(can measure error, cost, profit, ...).

The decision variables are subject to *constraints*.
The paradigm can be modelled mathematically.

The Optimization Paradigm, mathematically

Minimize $f(x_0, \ldots, x_{n-1})$
subject to

$$\begin{aligned} g_0(x_0, \ldots, x_{n-1}) &\leq 0 \\ &\ldots \\ g_{m-1}(x_0, \ldots, x_{n-1}) &\leq 0 \end{aligned}$$

obj. function	constraints	derivatives	
linear	linear	N	LP
non-linear	—	N	Moore-Skelboe
non-linear	—	Y	enum. crit. points
non-linear	equalities	Y	Lagrange
non-linear	inequalities	Y	KKT, John
non-linear	inequalities	N	CDGO

Stages for development of optimization algorithm:

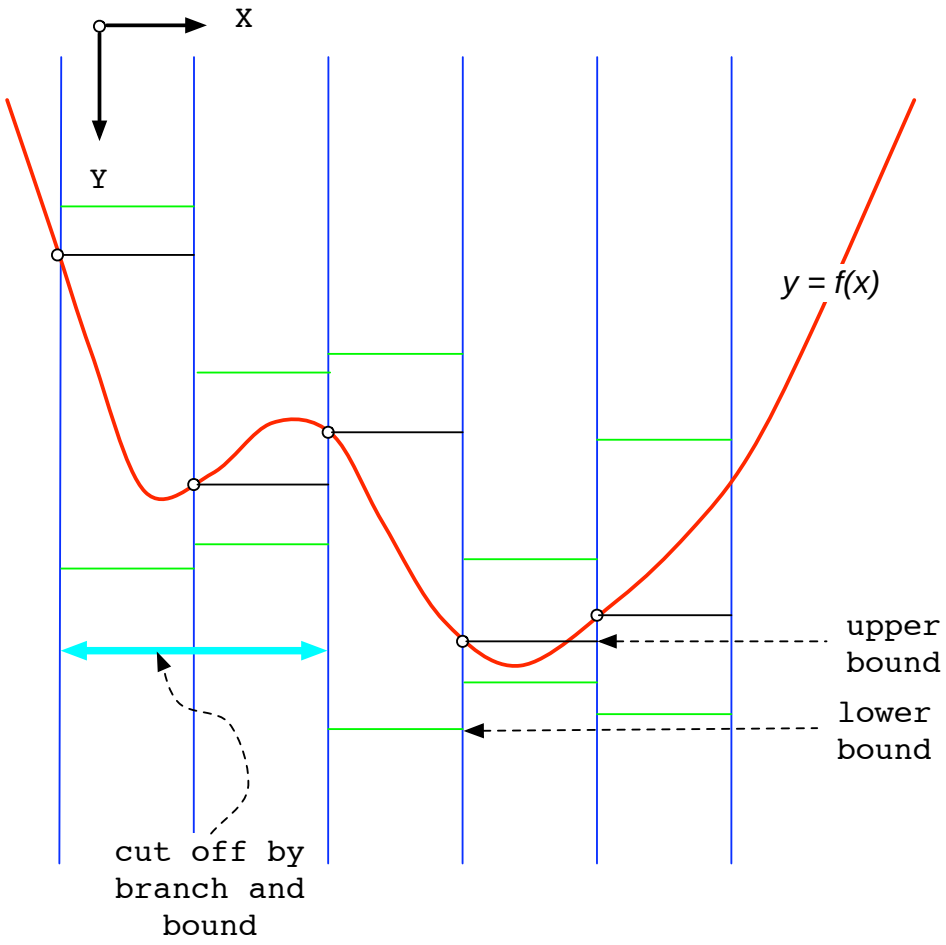
1. No use of derivatives; interval arithmetic

2. Convert to interval constraints

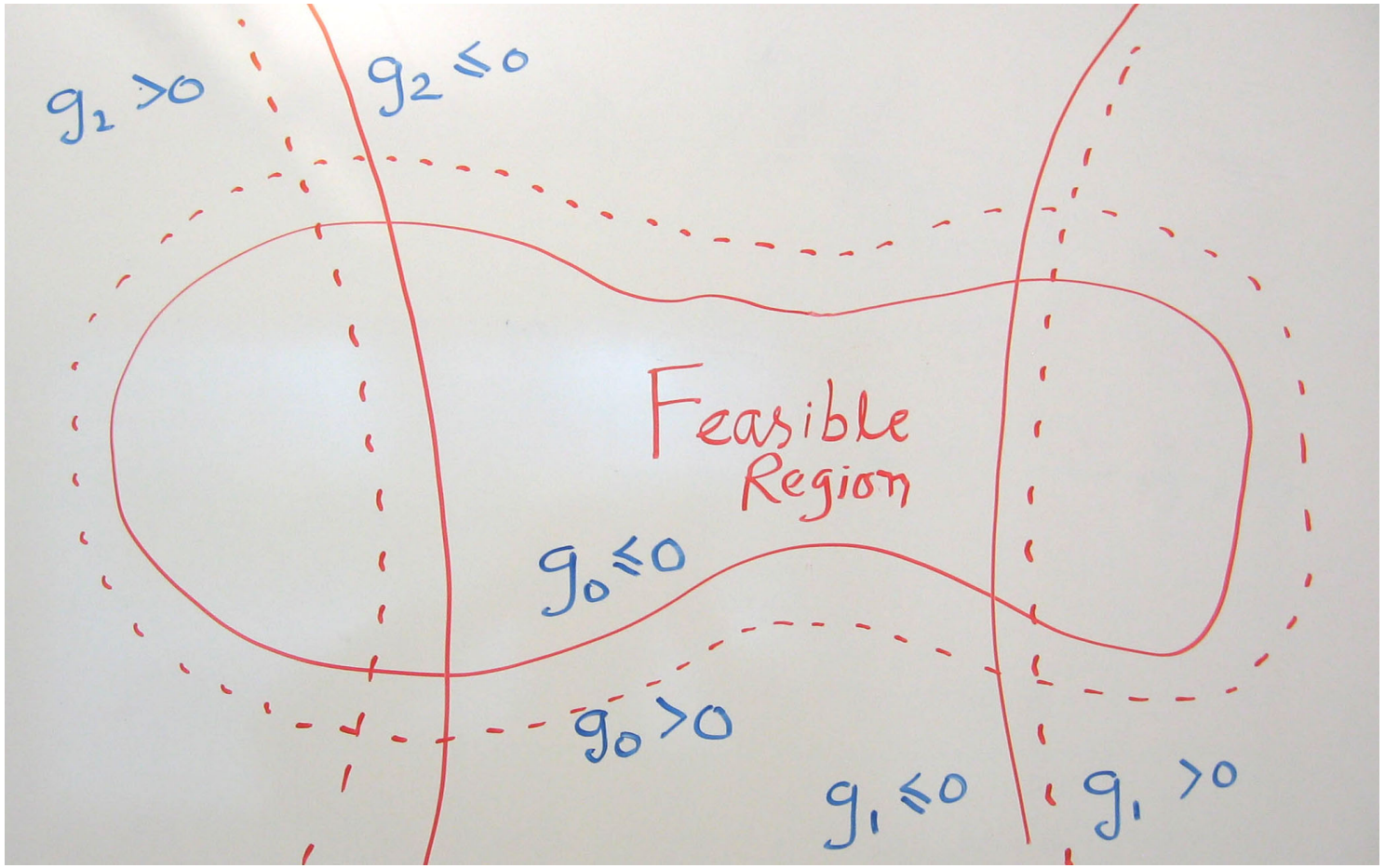
3. Add as redundant constraints KKT or John conditions

This talk: only first stage.

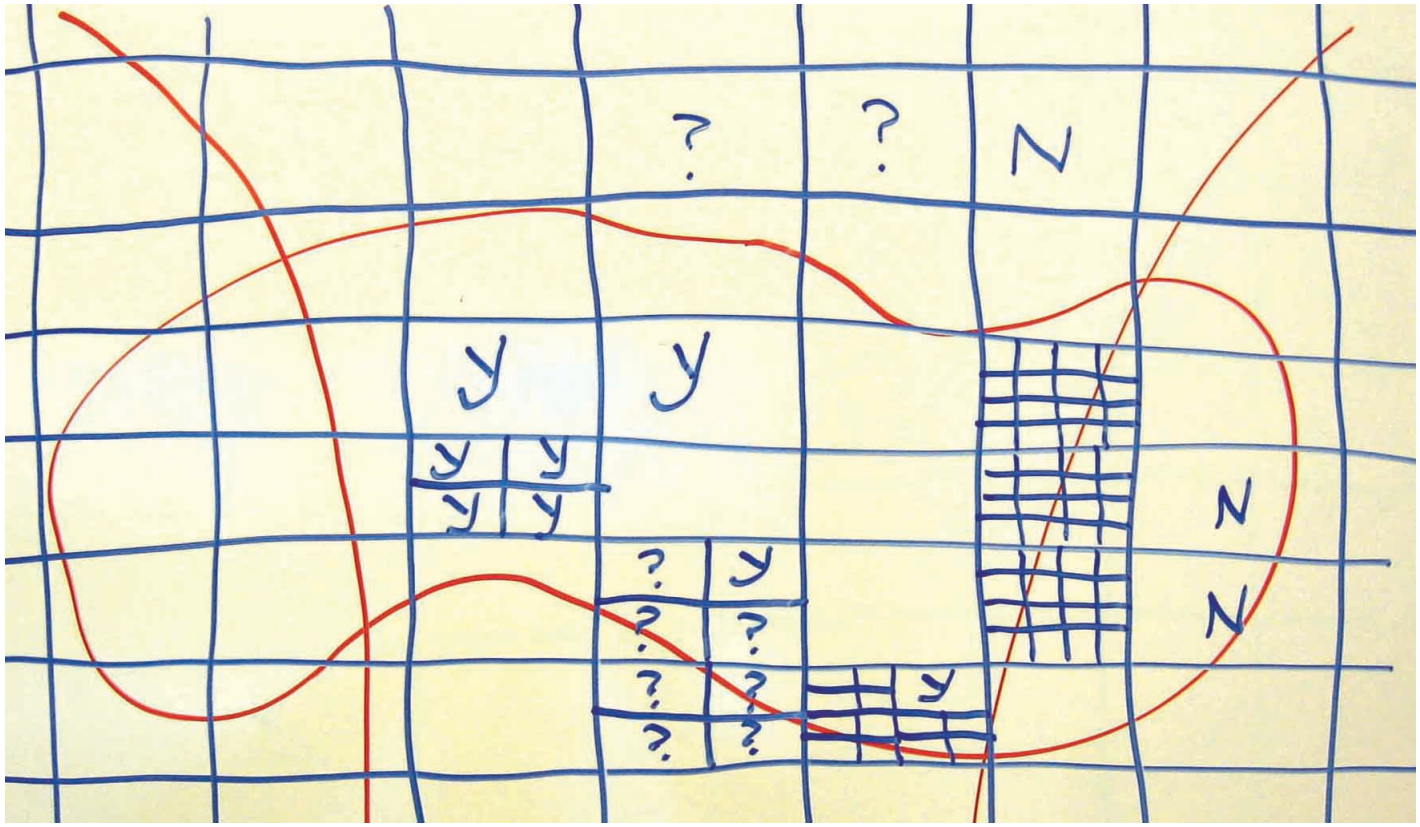
Non-linear Objective Function:



Non-linear Constraints:



(Heuristic) SIVIA for the constraints



Two lists of boxes: Y, ? (Ns are discarded)

CDGO: Constraint-Driven Global Optimization

Y-list: Contained in interior of feasible set. Allows unconstrained optimization. Gives upper bound to cut off boxes of both types. Can use (dynamic) Moore-Skelboe or point method. Gives lower bound on interior.

?-list: Contains boundary of feasible set. Only atomic boxes. Only need to store one. Gives lower bound on boundary if ?-list contains a feasible point.

Equality constraints

So far only inequality constraints $g_i(x_0, \ldots, x_{n-1}) \leq 0$.

If equality, then feasible set has no interior, numerically.
That is, Y-list is empty.

Equality in practice: $|g_i(x_0, \ldots, x_{n-1})| \leq \epsilon > 0$.

Of course, Interval Newton gives exact treatment of equality constraints.

Future work: incorporate Interval Newton into SIVIA.

Contributions of this work

- Verified bounds for global optimum of non-linear objective function subject to non-linear inequality constraints.
- Dynamic Moore-Skelboe (add boxes on the fly).
- Heuristic SIVIA (control processing order of boxes).